# PAPER 2

## PART – I : CHEMISTRY

# M2

## SECTION – I (Single Correct Choice Type)

1. The complex showing a spin-only magnetic moment of 2.82 B.M. is

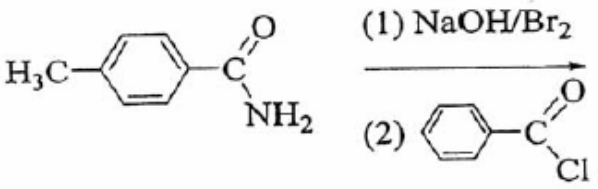
   A) $Ni(CO)_4$ B) $[NiCl_4]^{2-}$ C) $Ni(PPh_3)_4$ D) $[Ni(CN)_4]^{2-}$

**ANSWER: B**

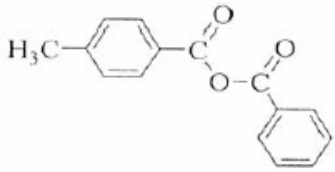
2. The species having pyramidal shape is

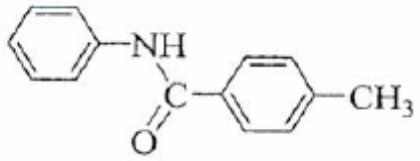
   A) $SO_3$ B) $BrF_3$ C) $SiO_3^{2-}$ D) $OSF_2$

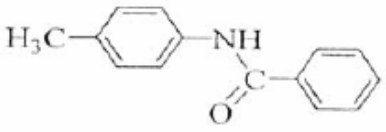
**ANSWER: D**

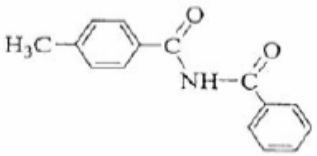
3. In the reaction 

the structure of the Product T is

A) 

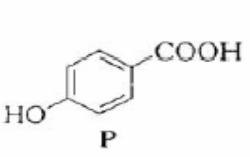

B) 


C) 

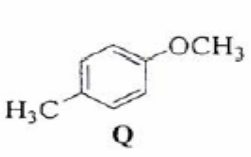

D) 


**ANSWER: C**

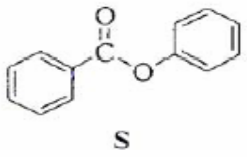
4. The compounds P, Q and S



P

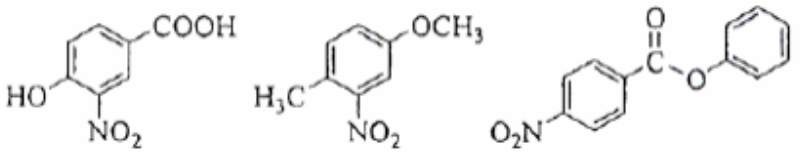


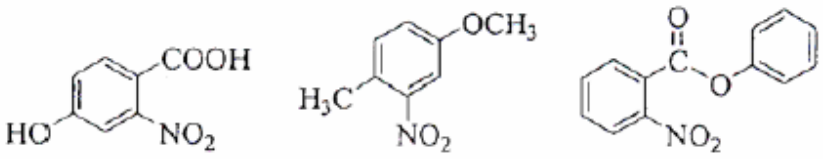
Q

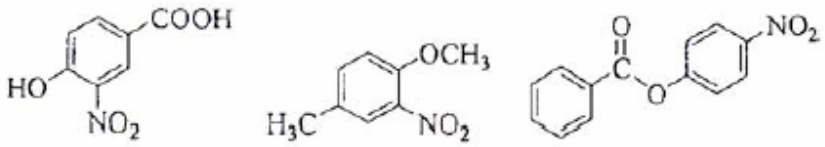


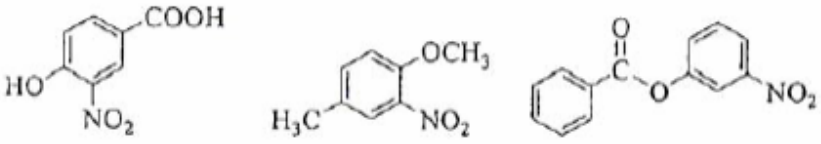
S

were separately subjected to nitration using $HNO_3/H_2SO_4$ mixture. The major product formed in each case respectively, is

A) 

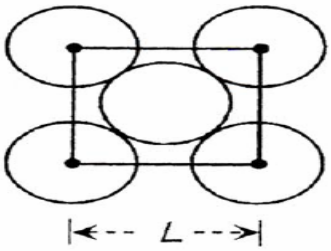

B) 


C) 


D) 


**ANSWER: C**

5. The packing efficiency of the two-dimensional square unit cell shown below is




A) 39.27% B) 68.02% C) 74.05% D) 78.54%

**ANSWER: D**

6. Assuming that Hund's rule is violated, the bond order and magnetic nature of the diatomic molecule $B_2$ is

A) 1 and diamagnetic
B) 0 and diamagnetic
C) 1 and paramagnetic
D) 0 and paramagnetic

**ANSWER: A**

## SECTION – II (Integer Type)

7. The total number of diprotic acids among the following is

$H_3PO_4$ $H_2SO_4$ $H_3PO_3$ $H_2CO_3$ $H_2S_2O_7$
$H_3BO_3$ $H_3PO_2$ $H_2CrO_4$ $H_2SO_3$

**ANSWER: 6**

8. Total number of geometrical isomers for the complex $[RhCl(CO)(PPh_3)(NH_3)]$ is

**ANSWER: 3**

9. Among the following, the number of elements showing only one non-zero oxidation state is

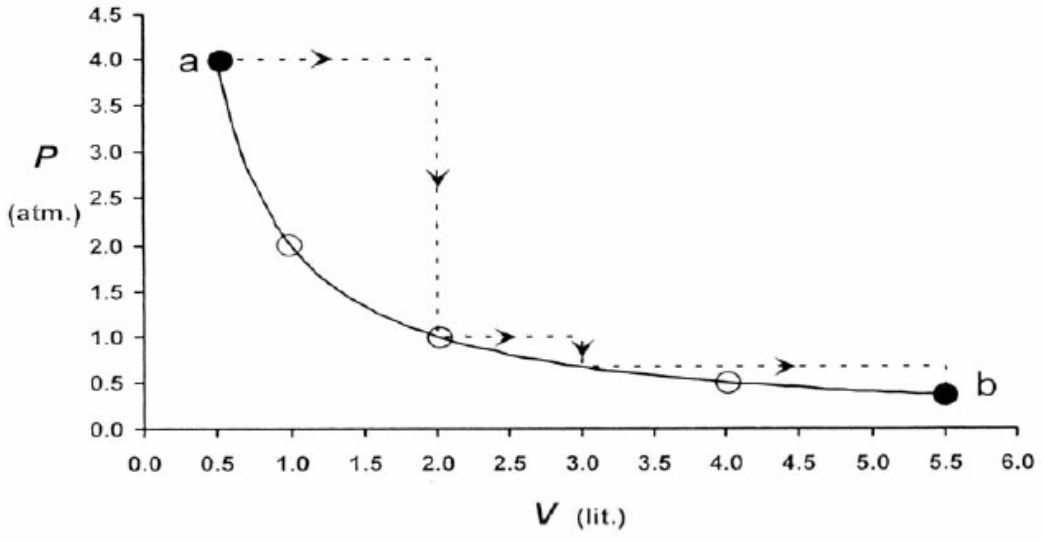
O, Cl, F, N, P, Sn, Tl, Na, Ti

**ANSWER: 2**

10. Silver (atomic weight = 108 g $mol^{-1}$) has a density of 10.5 g $cm^{-3}$. The number of silver atoms on a surface of area $10^{-12}$ $m^2$ can be expressed in scientific notation as $y \times 10^x$. The value of x is

**ANSWER: 7**

11. One mole of an ideal gas is taken from **a** to **b** along two paths denoted by the solid and the dashed lines as shown in the graph below. If the work done along the solid line path is $w_s$ and that along the dotted line path is $w_d$, then the integer closest to the ratio $w_d/w_s$ is

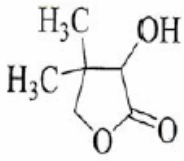



**ANSWER: 2**

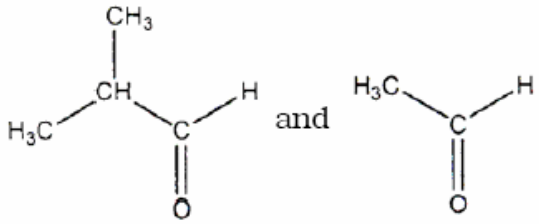
## SECTION – III (Paragraph Type)

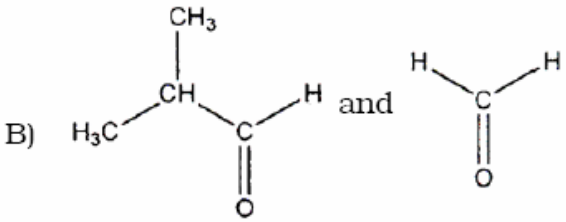
**Paragraph for Questions 12 to 14.**

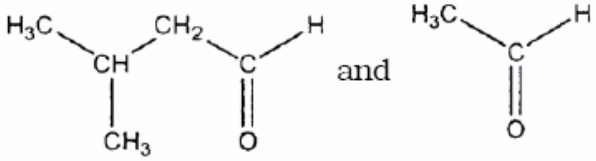
Two aliphatic aldehydes P and Q react in the presence of aqueous $K_2CO_3$ to give compound R, which upon treatment with HCN provides compound S. On acidification and heating, S gives the product shown below :

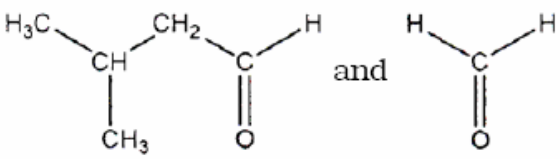


12. The compounds P and Q respectively are

A) 

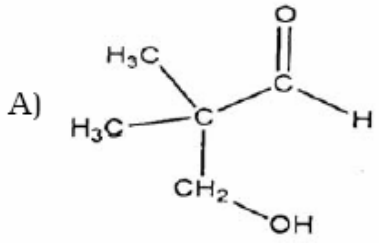
B) 

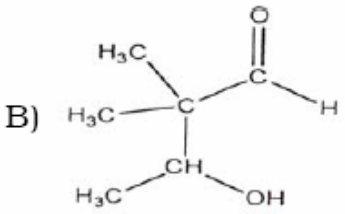
C) 

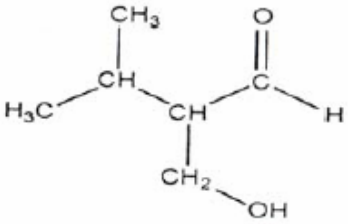
D) 

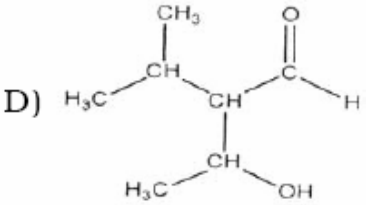
**ANSWER: B**

13. The compound R is

A) 

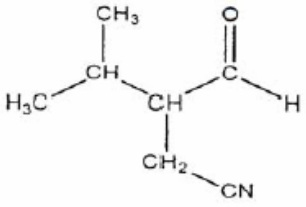
B) 

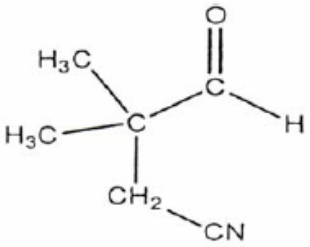
C) 

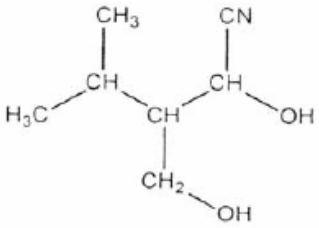
D) 

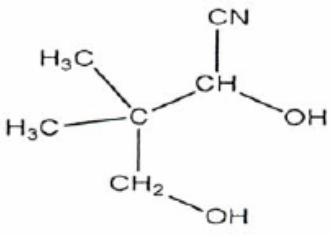
**ANSWER: A**

14. The compound S is

A) 

B) 

C) 

D) 

**ANSWER: D**

**Paragraph for Questions 15 to 17.**

The hydrogen-like species $Li^{2+}$ is in a spherically symmetric state $S_1$ with one radial node. Upon absorbing light the ion undergoes transition to a state $S_2$. The state $S_2$ has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

15. The state $S_1$ is

A) 1s B) 2s C) 2p D) 3s

**ANSWER: B**

16. Energy of the state $S_1$ in units of the hydrogen atom ground state energy is

A) 0.75 B) 1.50 C) 2.25 D) 4.50

**ANSWER: C**

17. The orbital angular momentum quantum number of the state $S_2$ is

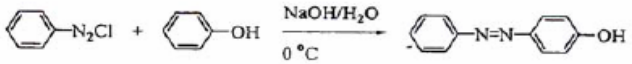
A) 0 B) 1 C) 2 D) 3

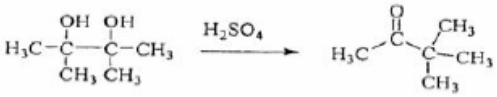
**ANSWER: B**

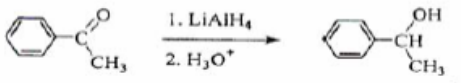
## SECTION – IV (Matrix Type)

18. Match the reactions in Column I with appropriate options in Column II.

**Column I**

A) 


B) 


C) 


D) 


**Column II**

p) Racemic mixture

q) Addition reaction

r) Substitution reaction

s) Coupling reaction

t) Carbocation intermediate

**ANSWER:**

**A: r and s**
**B: t**
**C: p and q**
**D: r**

19. All the compounds listed in **Column I** react with water. Match the result of the respective reactions with the appropriate options listed in **Column II**.

| **Column I** | **Column II** |
|---|---|
| A) $(CH_3)_2SiCl_2$ | p) Hydrogen halide formation |
| B) $XeF_4$ | q) Redox reaction |
| C) $Cl_2$ | r) Reacts with glass |
| D) $VCl_5$ | s) Polymerization |
| | t) $O_2$ formation |

**ANSWER:** **A: p and s**
**B: p and q and r and t**
**C: p and q**
**D: p**

## PART – II : MATHEMATICS

### SECTION – I (Single Correct Choice Type)

20. For r = 0, 1, ..., 10, let $A_r$, $B_r$ and $C_r$ denote, respectively, the coefficient of $x^r$ in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then

$$\sum_{r=1}^{10} A_r \left(B_{10}B_r - C_{10}A_r\right)$$

is equal to

A) $B_{10} - C_{10}$ B) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$ C) 0 D) $C_{10} - B_{10}$

**ANSWER: D**

21. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to

A) 25 B) 34 C) 42 D) 41

**ANSWER: D**

22. Let f be a real-valued function defined on the interval (–1, 1) such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1}\, dt$, for all $x \in (-1, 1)$, and let $f^{-1}$ be the inverse function of f. Then $(f^{-1})'(2)$ is equal to

A) 1 B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{e}$

**ANSWER: B**

23. If the distance of the point P(1, –2, 1) from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is

A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$ C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

**ANSWER: A**

24. Two adjacent sides of a parallelogram ABCD are given by

$\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$

The side AD is rotated by an acute angle $\alpha$ in the plane of the parallelogram so that AD becomes $AD'$. If $AD'$ makes a right angle with the side AB, then the cosine of the angle $\alpha$ is given by

A) $\frac{8}{9}$ B) $\frac{\sqrt{17}}{9}$ C) $\frac{1}{9}$ D) $\frac{4\sqrt{5}}{9}$

**ANSWER: B**

25. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

A) $\frac{3}{5}$ B) $\frac{6}{7}$ C) $\frac{20}{23}$ D) $\frac{9}{20}$

**ANSWER: C**

## SECTION – II (Integer Type)

26. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of [k] is

[**Note** : [k] denotes the largest integer less than or equal to k]

**ANSWER: 3**

27. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then $r^2$ is equal to

**ANSWER: 3**

28. Let f be a function defined on **R** (the set of all real numbers) such that

$f'(x)=2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$, for all $x \in \mathbf{R}$.

If g is a function defined on **R** with values in the interval $(0, \infty)$ such that

$$f(x) = \ell n\ (g(x)), \text{ for all } x \in \mathbf{R},$$

then the number of points in **R** at which g has a local maximum is

**ANSWER: 1**

29. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying

$a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$.

If $\frac{a_1^2+a_2^2+\ldots+a_{11}^2}{11}=90$, then the value of $\frac{a_1+a_2+\ldots+a_{11}}{11}$ is equal to

**ANSWER: 0**

30. Let k be a positive real number and let

$$A=\begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B=\begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}.$$

If det (adj A) + det(adj B) $=10^6$, then [k] is equal to

[**Note** : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

**ANSWER: 4**

## SECTION – III (Paragraph Type)

**Paragraph for questions 31 to 33.**

Consider the polynomial

$f(x)=1+2x+3x^2+4x^3$.

Let s be the sum of all distinct real roots of f(x) and let $t = |s|$.

31. The real number s lies in the interval

A) $\left(-\frac{1}{4},0\right)$ B) $\left(-11,-\frac{3}{4}\right)$ C) $\left(-\frac{3}{4},-\frac{1}{2}\right)$ D) $\left(0,\frac{1}{4}\right)$

**ANSWER: C**

32. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

A) $\left(\frac{3}{4},3\right)$ B) $\left(\frac{21}{64},\frac{11}{16}\right)$ C) $(9,10)$ D) $\left(0,\frac{21}{64}\right)$

**ANSWER: A**

33. The function f′(x) is

A) increasing in $\left(-t,-\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4},t\right)$

B) decreasing in $\left(-t,-\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4},t\right)$

C) increasing in (–t, t)

D) decreasing in (–t, t)

**ANSWER: B**

**Paragraph for Questions 34 to 36.**

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B.

34. The coordinates of A and B are

A) (3, 0) and (0, 2)

B) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5},\frac{8}{5}\right)$

C) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and (0, 2)

D) (3, 0) and $\left(-\frac{9}{5},\frac{8}{5}\right)$

**ANSWER: D**

35. The orthocenter of the triangle PAB is

A) $\left(5,\frac{8}{7}\right)$ B) $\left(\frac{7}{5},\frac{25}{8}\right)$ C) $\left(\frac{11}{5},\frac{8}{5}\right)$ D) $\left(\frac{8}{25},\frac{7}{5}\right)$

**ANSWER: C**

36. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

**ANSWER: A**

## SECTION – IV (Matrix Type)

37. Match the statements in **Column-I** with those in **Column-II**.

[**Note**: Here z takes values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z.]

| **Column I** | **Column II** |
|---|---|
| A) The set of points z satisfying $\lvert z-i\rvert z\rVert = \lvert z+i\rvert z\rvert$ is contained in or equal to | p) an ellipse with eccentricity $\frac{4}{5}$ |
| | q) the set of points z satisfying Im z = 0 |
| B) The set of points z satisfying $\lvert z+4\rvert + \lvert z-4\rvert = 10$ is contained in or equal to | r) the set of points z satisfying $\lvert \text{Im } z\rvert \le 1$ |
| C) If $\lvert w\rvert = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to | s) the set of points z satisfying $\lvert \text{Re } z\rvert \le 2$ |
| D) If $\lvert w\rvert = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to | t) the set of points z satisfying $\lvert z\rvert \le 3$ |

**ANSWER:**

**A: q and r**
**B: p**
**C: p and s and t**
**D: q and r and s and t**

38. Match the statements in **Column-I** with the values in **Column-II**.

| **Column I** | **Column II** |
|---|---|
| A) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then $d^2$ is | p) – 4 |
| B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are | q) 0 |
| C) Non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b}=0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of $\mu$ are | r) 4 |
| D) Let f be the function on $[-\pi,\pi]$ given by f(0) = 9 and f(x) = $\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x\neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is | s) 5 |
| | t) 6 |

**ANSWER:**

**A: t**
**B: p and r**
**C: either q or (q and s)**
**D: r**

## PART – III : PHYSICS

## SECTION – I (Single Correct Choice Type)

39. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is

A) 0.02 mm B) 0.05 mm C) 0.1 mm D) 0.2 mm

**ANSWER: D**

40. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 $ms^{-1}$, the mass of the string is

A) 5 grams B) 10 grams C) 20 grams D) 40 grams

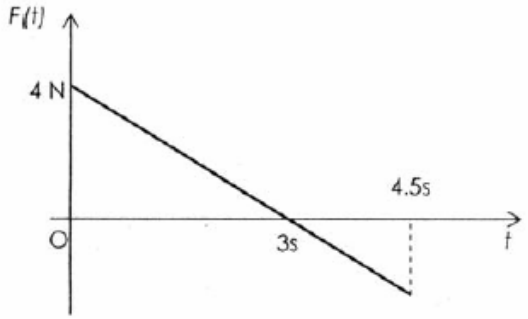
**ANSWER: B**

41. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

A) virtual and at a distance of 16 cm from the mirror
B) real and at a distance of 16 cm from the mirror
C) virtual and at a distance of 20 cm from the mirror
D) real and at a distance of 20 cm from the mirror

**ANSWER: B**

42. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with $t$ as shown in the figure. The kinetic energy of the block after 4.5 seconds is




A) 4.50 J B) 7.50 J C) 5.06 J D) 14.06 J

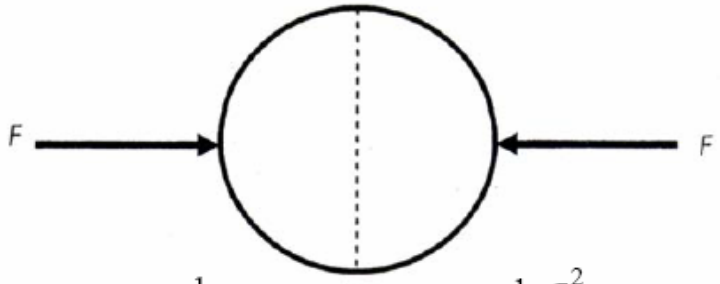
**ANSWER: C**

43. A tiny spherical oil drop carrying a net charge $q$ is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times 10^5$ $Vm^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3}$ m $s^{-1}$. Given g = 9.8 m $s^{-2}$, viscosity of the air = $1.8 \times 10^{-5}$ Ns $m^{-2}$ and the density of oil = 900 kg $m^{-3}$, the magnitude of $q$ is

A) $1.6 \times 10^{-19}$ C B) $3.2 \times 10^{-19}$ C C) $4.8 \times 10^{-19}$ C D) $8.0 \times 10^{-19}$ C

**ANSWER: D**

44. A uniformly charged thin spherical shell of radius $R$ carries uniform surface charge density of $\sigma$ per unit area. It is made of two hemispherical shells, held together by pressing them with force $F$ (see figure). $F$ is proportional to




A) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ B) $\frac{1}{\varepsilon_0}\sigma^2 R$ C) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ D) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

**ANSWER: A**

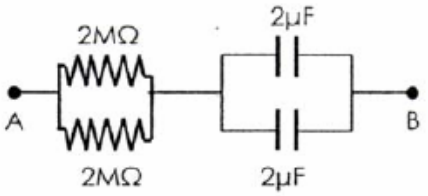
## SECTION – II (Integer Type)

45. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. In the initial temperature of the gas is $T_i$ (in Kelvin) and the final temperature is $aT_i$, the value of $a$ is

**ANSWER: 4**

46. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V ?

[Take : $\ell n\ 5 = 1.6$, $\ell n\ 3 = 1.1$]




**ANSWER: 2**

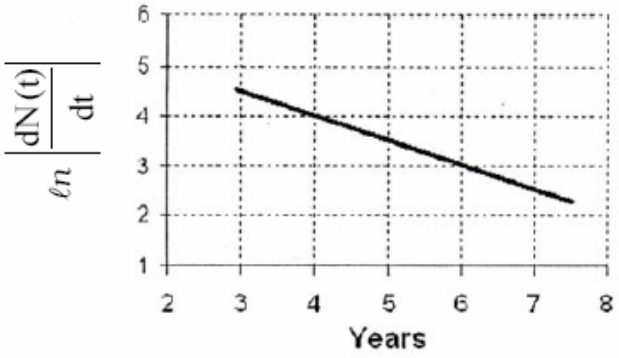
47. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour ?

**ANSWER: 3**

48. A large glass slab $(\mu = 5/3)$ of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius $R$ cm. What is the value of $R$ ?

**ANSWER: 6**

49. To determine the half life of a radioactive element, a student plots a graph of $\ell n \left|\frac{dN(t)}{dt}\right|$ versus $t$. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time $t$. If the number of radioactive nuclei of this element decreases by a factor of $p$ after 4.16 years, the value of $p$ is




**ANSWER: 8**

## SECTION – III (Paragraph Type)

**Paragraph for questions 50 to 52.**

When liquid medicine of density $\rho$ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension $T$ when the radius of the drop is $R$. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

50. If the radius of the opening of the dropper is $r$, the vertical force due to the surface tension on the drop of radius $R$ (assuming $r << R$) is

A) $2\pi rT$ B) $2\pi RT$ C) $\frac{2\pi r^2 T}{R}$ D) $\frac{2\pi R^2 T}{r}$

**ANSWER: C**

51. If $r = 5\times10^{-4}\,m, \rho = 10^{3}\,kgm^{-3}, g = 10\ ms^{-2}, T = 0.11\,Nm^{-1}$, the radius of the drop when it detaches from the dropper is approximately

A) $1.4\times10^{-3}\,m$ B) $3.3\times10^{-3}\,m$ C) $2.0\times10^{-3}\,m$ D) $4.1\times10^{-3}\,m$

**ANSWER: A**

52. After the drop detaches, its surface energy is

A) $1.4\times10^{-6}J$ B) $2.7\times10^{-6}J$ C) $5.4\times10^{-6}J$ D) $8.1\times10^{-6}J$

**ANSWER: B**

**Paragraph for Questions 53 to 55.**

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

53. A diatomic molecule has moment of inertia *I*. By Bohr's quantization condition its rotational energy in the $n^{th}$ level ($n$ = 0 is not allowed) is

A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$ C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

**ANSWER: D**

54. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to (Take $h = 2\pi\times 10^{-34}$ J s)

A) $2.76\times 10^{-46}$ kg m$^2$
B) $1.87\times 10^{-46}$ kg m$^2$
C) $4.67\times 10^{-47}$ kg m$^2$
D) $1.17\times 10^{-47}$ kg m$^2$

**ANSWER: B**

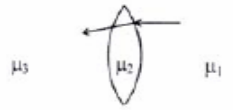
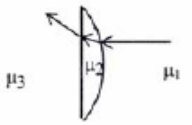
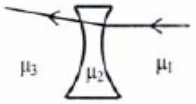
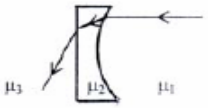
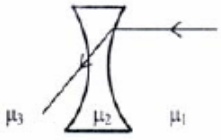
55. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where 1 a.m.u. $=\frac{5}{3}\times 10^{-27}$ kg, is close to

A) $2.4\times 10^{-10}$ m
B) $1.9\times 10^{-10}$ m
C) $1.3\times 10^{-10}$ m
D) $4.4\times 10^{-11}$ m

**ANSWER: C**

## SECTION – IV (Matrix Type)

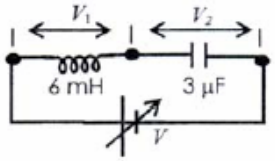
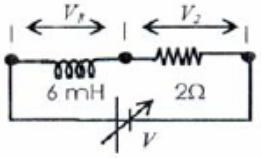
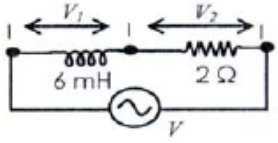
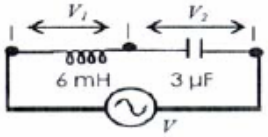
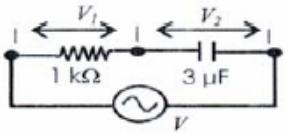
56. Two transparent media of refractive indices $\mu_1$ and $\mu_3$ have a solid lens shaped transpar material of refractive index $\mu_2$ between them as shown in figures in **Column II**. A traversing these media is also shown in the figures. In **Column I** different relationsh between $\mu_1, \mu_2$ and $\mu_3$ are given. Match them to the ray diagrams shown in **Column**

| **Column I** | **Column II** |
|---|---|
| A) $\mu_1 < \mu_2$ | p)  |
| B) $\mu_1 > \mu_2$ | q)  |
| C) $\mu_2 = \mu_3$ | r)  |
| D) $\mu_2 > \mu_3$ | s)  |
| | t)  |

**ANSWER:**

**A: p and r**
**B: q and s and t**
**C: p and r and t**
**D: q and s**

57. You are given many resistances, capacitors and inductors. These are connected to variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequenc (the next three circuits) in different ways as shown in **Column II.** When a current (steady state for DC or *rms* for AC) flows through the circuit, the corresponding voltage V and $V_2$. (indicated in circuits) are related as shown in **column I.** Match the two

| **Column I** | **Column II** |
|---|---|
| A) $I \neq 0, V_1$ is proportional to $I$ | p)   |
| B) $I \neq 0, V_2 > V_1$ | q)   |
| C) $V_1 = 0, V_2 = V$ | r)   |
| D) $I \neq 0, V_2$ is proportional to $I$ | s)   |
| | t)   |

**ANSWER:**

**A: r and s and t**
**B: q and r and s and t**
**C: p and q**
**D: q and r and s and t**

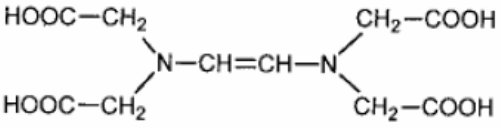
**********

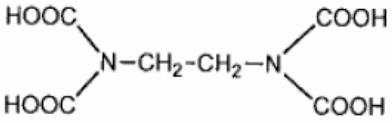
## PART – I : CHEMISTRY

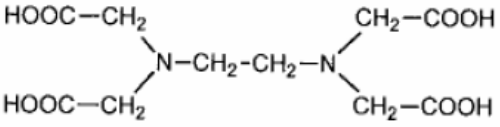
### SECTION – I (Single Correct Choice Type)

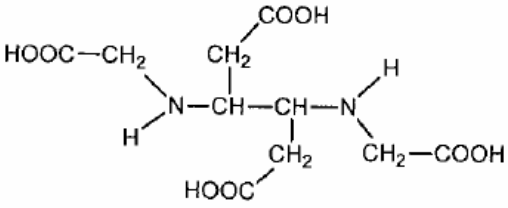
This Section contains **8 multiple choice questions**. Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

1. The correct structure of ethylenediaminetetraacetic acid (EDTA) is

A) 

B) 

C) 

D) 

**ANSWER: C**

2. The ionization isomer of $[Cr(H_2O)_4Cl(NO_2)]Cl$ is

A) $[Cr(H_2O)_4(O_2N)]Cl_2$

B) $[Cr(H_2O)_4Cl_2](NO_2)$

C) $[Cr(H_2O)_4Cl(ONO)]Cl$

D) $[Cr(H_2O)_4Cl_2(NO_2)]\cdot H_2O$

**ANSWER: B**

3. The synthesis of 3-octyne is achieved by adding a bromoalkane into a mixture of sodium amide and an alkyne. The bromoalkane and alkyne respectively are

A) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3CH_2C\equiv CH$

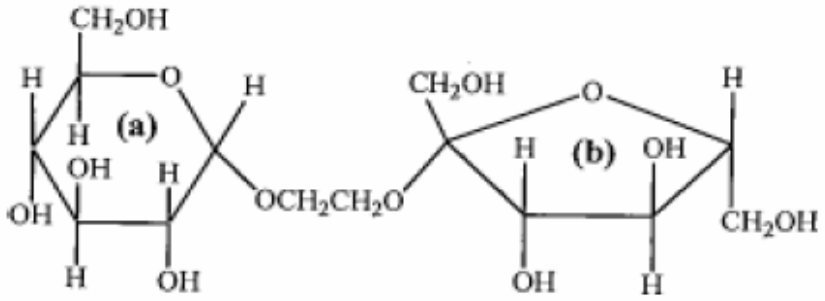
B) $BrCH_2CH_2CH_3$ and $CH_3CH_2CH_2C\equiv CH$

C) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3C\equiv CH$

D) $BrCH_2CH_2CH_2CH_3$ and $CH_3CH_2C\equiv CH$

**ANSWER: D**

4. The correct statement about the following disaccharide is



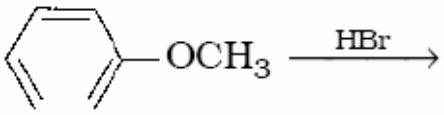

A) Ring **(a)** is pyranose with $\alpha$-glycosidic link

B) Ring **(a)** is furanose with $\alpha$-glycosidic link

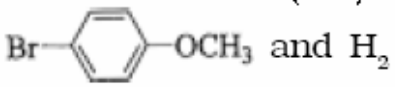
C) Ring **(b)** is furanose with $\alpha$-glycosidic link

D) Ring **(b)** is pyranose with $\beta$-glycosidic link

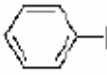
**ANSWER: A**

5. In the reaction  the products are



A)  and $H_2$



B)  and $CH_3Br$



C)  and $CH_3OH$

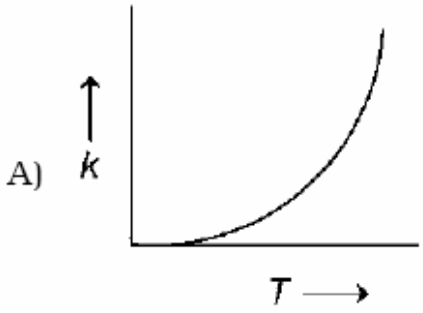


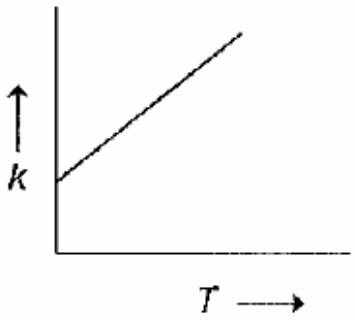
D)  and $CH_3Br$

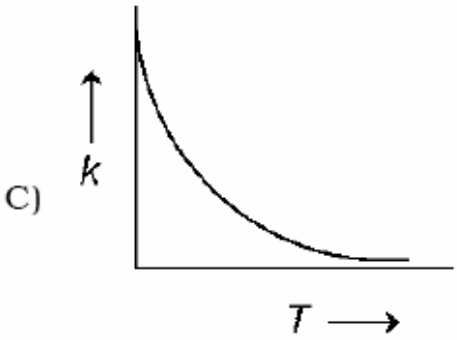


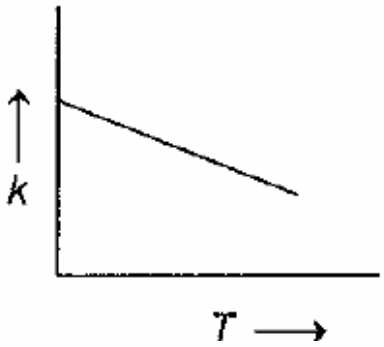
**ANSWER: D**

6. Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that follows Arrhenius equation is

A) 


B) 


C) 


D) 


**ANSWER: A**

7. The species which by definition has **ZERO** standard molar enthalpy of formation at 298 K is

A) $Br_2$ (g) B) $Cl_2$ (g) C) $H_2O$ (g) D) $CH_4$ (g)

**ANSWER: B**

8. The bond energy (in **kcal mol$^{-1}$**) of a C–C single bond is approximately

A) 1 B) 10 C) 100 D) 1000

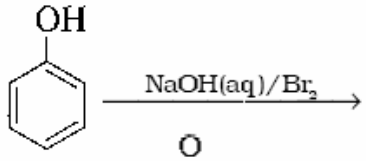
**ANSWER: C**

## SECTION – II (Multiple Correct Choice Type)

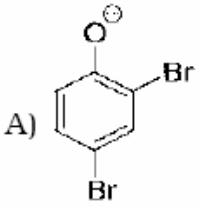
9. The reagent(s) used for softening the temporary hardness of water is(are)

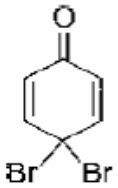
A) $Ca_3(PO_4)_2$ B) $Ca(OH)_2$ C) $Na_2CO_3$ D) NaOCl

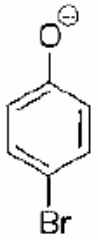
**ANSWER: B**

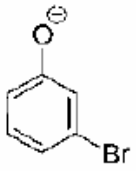
10. In the reaction 

the intermediate(s) is(are)

A) 

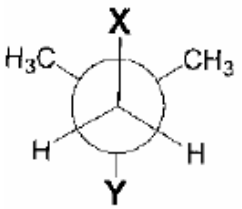

B) 


C) 


D) 


**ANSWER: A and C**

11. In the Newman projection for 2,2-dimethylbutane




**X** and **Y** can respectively be

A) H and H B) H and $C_2H_5$

C) $C_2H_5$ and H D) $CH_3$ and $CH_3$

**ANSWER: B and D**

12. Among the following, the intensive property is (properties are)

A) molar conductivity B) electromotive force

C) resistance D) heat capacity

**ANSWER: A and B**

13. Aqueous solutions of $HNO_3$, KOH, $CH_3COOH$, and $CH_3COONa$ of identical concentrations are provided. The pair(s) of solutions which form a buffer upon mixing is(are)

A) $HNO_3$ and $CH_3COOH$
B) KOH and $CH_3COONa$
C) $HNO_3$ and $CH_3COONa$
D) $CH_3COOH$ and $CH_3COONa$

**ANSWER: C and D**

## SECTION – III (Paragraph Type)

**Paragraph for Questions 14 to 16**

Copper is the most noble of the first row transition metals and occurs in small deposits in several countries. Ores of copper include chalcanthite ($CuSO_4 \cdot 5H_2O$), atacamite ($Cu_2Cl(OH)_3$), cuprite ($Cu_2O$), copper glance ($Cu_2S$) and malachite ($Cu_2(OH)_2CO_3$). However, 80% of the world copper production comes from the ore chalcopyrite ($CuFeS_2$). The extraction of copper from chalcopyrite involves partial roasting, removal of iron and self-reduction.

14. Partial roasting of chalcopyrite produces

A) $Cu_2S$ and FeO
B) $Cu_2O$ and FeO
C) CuS and $Fe_2O_3$
D) $Cu_2O$ and $Fe_2O_3$

**ANSWER: A**

15. Iron is removed from chalcopyrite as

A) FeO
B) FeS
C) $Fe_2O_3$
D) $FeSiO_3$

**ANSWER: D**

16. In self-reduction, the reducing species is

A) S  B) $O^{2-}$

C) $S^{2-}$  D) $SO_2$

**ANSWER: C**

**Paragraph for Questions 17 to 18**

The concentration of potassium ions inside a biological cell is at least twenty times higher than the outside. The resulting potential difference across the cell is important in several processes such as transmission of nerve impulses and maintaining the ion balance. A simple model for such a concentration cell involving a metal M is:

$M(s)|M^+(aq;\ 0.05\ molar)||M^+(aq;\ 1\ molar)|M(s)$

For the above electrolytic cell the magnitude of the cell potential $|E_{cell}| = 70$ mV.

17. For the above cell

A) $E_{cell} < 0;\ \Delta G > 0$  B) $E_{cell} > 0;\ \Delta G < 0$

C) $E_{cell} < 0;\ \Delta G^0 > 0$  D) $E_{cell} > 0;\ \Delta G^0 < 0$

**ANSWER: B**

18. If the 0.05 molar solution of $M^+$ is replaced by a 0.0025 molar $M^+$ solution, then the magnitude of the cell potential would be

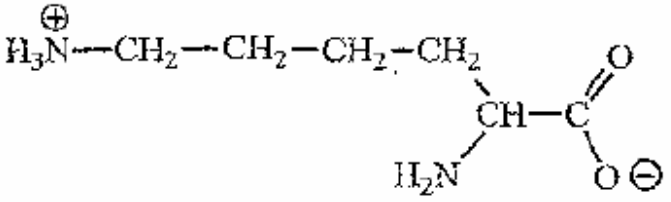
A) 35 mV  B) 70 mV

C) 140 mV  D) 700 mV

**ANSWER: C**

## SECTION – IV (Integer Type)

19. The total number of basic groups in the following form of lysine is

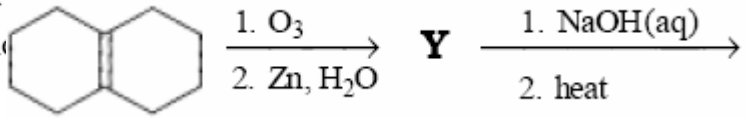


**ANSWER: 2**

20. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula $C_4H_6$ is

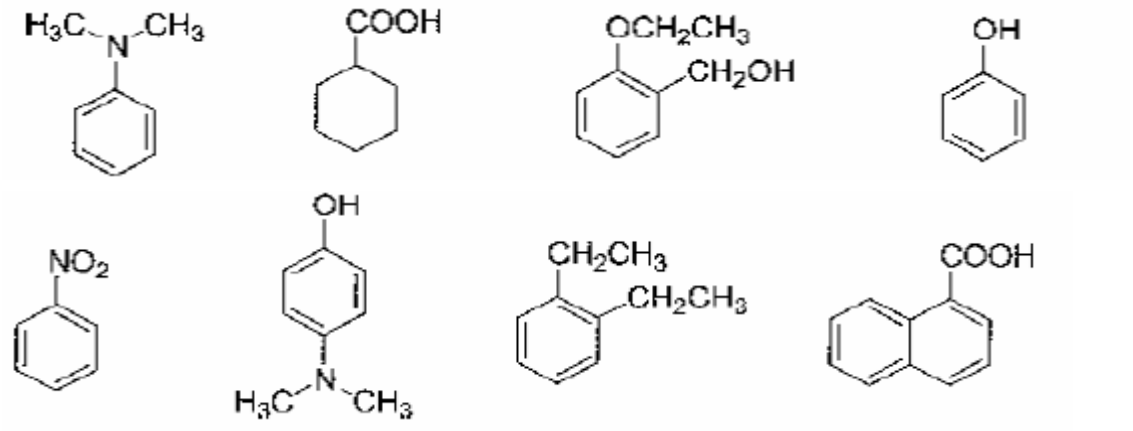
**ANSWER: 5**

21. In the ecular aldol condensation produ



**ANSWER: 1**

22. Amongst the following, the total number of compounds soluble in aqueous NaOH is



**ANSWER: 4**

23. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is

| | | | | |
|---|---|---|---|---|
| KCN | $K_2SO_4$ | $(NH_4)_2C_2O_4$ | NaCl | $Zn(NO_3)_2$ |
| $FeCl_3$ | $K_2CO_3$ | $NH_4NO_3$ | LiCN | |

**ANSWER: 3**

24. Based on VSEPR theory, the number of 90 degree F–Br–F angles in $BrF_5$ is

**ANSWER: either 0 or 8**

25. The value of n in the molecular formula $Be_nAl_2Si_6O_{18}$ is

**ANSWER: 3**

26. A student performs a titration with different burettes and finds titre values of 25.2 mL, 25.25 mL, and 25.0 mL. The number of significant figures in the average titre value is

**ANSWER: 3**

27. The concentration of R in the reaction R $\rightarrow$ P was measured as a function of time and the following data is obtained:

| [R] (molar) | 1.0 | 0.75 | 0.40 | 0.10 |
|---|---|---|---|---|
| t(min.) | 0.0 | 0.05 | 0.12 | 0.18 |

The order of the reaction is

**ANSWER: 0**

28. The number of neutrons emitted when $^{235}_{92}U$ undergoes controlled nuclear fission to $^{142}_{54}Xe$ and $^{90}_{38}Sr$ is

**ANSWER: 4**

## PART – II : MATHEMATICS

### SECTION – I (Single Correct Choice Type)

29. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$ is

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) 1 D) $\sqrt{3}$

**ANSWER: D**

30. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

A) $x + 2y - 2z = 0$ B) $3x + 2y - 2z = 0$ C) $x - 2y + z = 0$ D) $5x + 2y - 4z = 0$

**ANSWER: C**

31. Let $\omega$ be a complex cube root of unity with $\omega\neq 1$. A fair die is thrown three times. If $r_1$, $r_2$ and $r_3$ are the numbers obtained on the die, then the probability that $\omega^{r_1}+\omega^{r_2}+\omega^{r_3}=0$ is

A) $\frac{1}{18}$ B) $\frac{1}{9}$ C) $\frac{2}{9}$ D) $\frac{1}{36}$

**ANSWER: C**

32. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a

A) parallelogram, which is neither a rhombus nor a rectangle
B) square
C) rectangle, but not a square
D) rhombus, but not a square

**ANSWER: A**

33. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

A) 0 B) $2^9 - 1$ C) 168 D) 2

**ANSWER: A**

34. The value of $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t \, \ell n \, (1+t)}{t^4+4} \, dt$ is

A) 0 B) $\frac{1}{12}$ C) $\frac{1}{24}$ D) $\frac{1}{64}$

**ANSWER: B**

35. Let p and q be real numbers such that $p \neq 0, p^3 \neq q$ and $p^3 \neq -q$. If $\alpha$ and $\beta$ are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$ D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

**ANSWER: B**

36. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

A) a = b and c ≠ b B) a = c and a ≠ b

C) a ≠ b and c ≠ b D) a = b = c

**ANSWER: D**

## SECTION – II (Multiple Correct Choice Type)

37. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

A) $-\frac{1}{r}$ B) $\frac{1}{r}$ C) $\frac{2}{r}$ D) $-\frac{2}{r}$

**ANSWER: C and D**

38. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are)

A) $-\left(2+\sqrt{3}\right)$ B) $1+\sqrt{3}$ C) $2+\sqrt{3}$ D) $4\sqrt{3}$

**ANSWER: B**

39. Let $z_1$ and $z_2$ be two distinct complex numbers and let $z = (1 - t) z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number w, then

A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

B) $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

C) $\begin{vmatrix} z-z_1 & \bar{z}-\bar{z}_1 \\ z_2-z_1 & \bar{z}_2-\bar{z}_1 \end{vmatrix} = 0$

D) $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

**ANSWER: A and C and D**

40. Let f be a real-valued function defined on the interval $(0, \infty)$ by $f(x) = \ell n\ x + \int_0^x \sqrt{1+\sin t}\ dt$. Then which of the following statement(s) is (are) true ?

A) $f''(x)$ exists for all $x \in (0, \infty)$

B) $f'(x)$ exists for all $x \in (0, \infty)$ and $f'$ is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$

C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$

D) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \geq \beta$ for all $x \in (0, \infty)$

**ANSWER: B and C**

41. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is (are)

A) $\frac{22}{7} - \pi$ B) $\frac{2}{105}$ C) 0 D) $\frac{71}{15} - \frac{3\pi}{2}$

**ANSWER: A**

## SECTION – III (Paragraph Type)

### Paragraph for Questions 42 to 44

Let p be an odd prime number and $T_p$ be the following set of 2×2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots, p-1\} \right\}$$

42. The number of A in $T_p$ such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is

A) $(p-1)^2$ B) $2(p-1)$ C) $(p-1)^2 + 1$ D) $2p-1$

**ANSWER: D**

43. The number of A in $T_p$ such that the trace of A is not divisible by p but det (A) is divisible by p is

[**Note :** The trace of a matrix is the sum of its diagonal entries.]

A) $(p-1)(p^2-p+1)$ B) $p^3-(p-1)^2$ C) $(p-1)^2$ D) $(p-1)(p^2-2)$

**ANSWER: C**

44. The number of A in $T_p$ such that det (A) is not divisible by p is

A) $2p^2$ B) $p^3 - 5p$ C) $p^3 - 3p$ D) $p^3 - p^2$

**ANSWER: D**

**Paragraph for Questions 45 to 46**

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

45. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

A) $2x - \sqrt{5}y - 20 = 0$ B) $2x - \sqrt{5}y + 4 = 0$

C) $3x - 4y + 8 = 0$ D) $4x - 3y + 4 = 0$

**ANSWER: B**

46. Equation of the circle with AB as its diameter is

A) $x^2 + y^2 - 12x + 24 = 0$ B) $x^2 + y^2 + 12x + 24 = 0$

C) $x^2 + y^2 + 24x - 12 = 0$ D) $x^2 + y^2 - 24x - 12 = 0$

**ANSWER: A**

## SECTION – IV (Integer Type)

47. The number of values of $\theta$ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is

**ANSWER: 3**

48. The maximum value of the expression $\dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

**ANSWER: 2**

49. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \dfrac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \dfrac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a} + \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$ is

**ANSWER: 5**

50. The line $2x + y = 1$ is tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

**ANSWER: 2**

51. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is

**ANSWER: 6**

52. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by

$$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd,} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\dfrac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx$ is

**ANSWER: 4**

53. Let $\omega$ be the complex number $\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to

**ANSWER: 1**

54. Let $S_k$, $k = 1, 2, \ldots, 100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} \left|\left(k^2 - 3k + 1\right) S_k\right|$ is

**ANSWER: 3**

55. The number of all possible values of $\theta$, where $0 < \theta < \pi$, for which the system of equations

$$(y + z) \cos 3\theta = (xyz) \sin 3\theta$$

$$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz) \sin 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$$

have a solution $(x_0, y_0, z_0)$ with $y_0 z_0 \neq 0$, is

**ANSWER: 3**

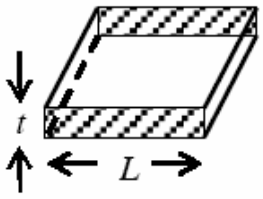
56. Let f be a real-valued differentiable function on **R** (the set of all real numbers) such that f(1) = 1. If the y-intercept of the tangent at any point P(x, y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(-3) is equal to

**ANSWER: 9**

## PART – III : PHYSICS

**SECTION – I** **(Single Correct Choice Type)**

57. Consider a thin square sheet of side L and thickness t, made of a material of resistivity $\rho$. The resistance between two opposite faces, shown by the shaded areas in the figure is




A) directly proportional to L
B) directly proportional to t
C) independent of L
D) independent of t

**ANSWER: C**

58. A real gas behaves like an ideal gas if its
A) pressure and temperature are both high
B) pressure and temperature are both low
C) pressure is high and temperature is low
D) pressure is low and temperature is high

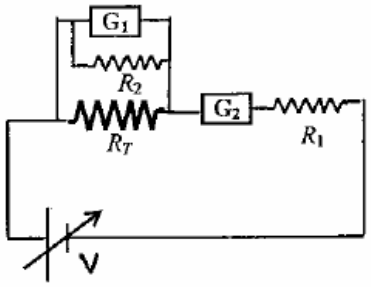
**ANSWER: D**

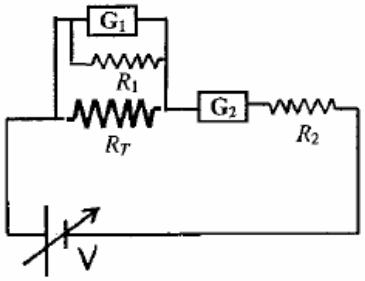
59. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances $R_{100}$, $R_{60}$ and $R_{40}$, respectively, the relation between these resistances is

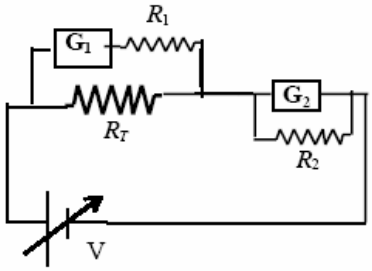
A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$
B) $R_{100} = R_{40} + R_{60}$
C) $R_{100} > R_{60} > R_{40}$
D) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

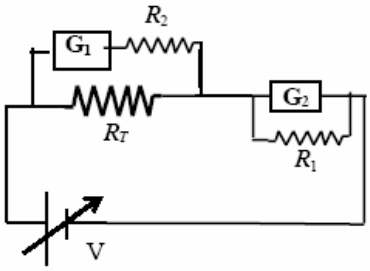
**ANSWER: D**

60. To verify Ohm's law, a student is provided with a test resistor $R_T$, a high resistance $R_1$, a small resistance $R_2$, two identical galvanometers $G_1$ and $G_2$, and a variable voltage source V. The correct circuit to carry out the experiment is

A)



B)



C)



D)



**ANSWER: C**

61. An AC voltage source of variable angular frequency $\omega$ and fixed amplitude $V_0$ is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When $\omega$ is increased

A) the bulb glows dimmer

B) the bulb glows brighter

C) total impedance of the circuit is unchanged

D) total impedance of the circuit increases

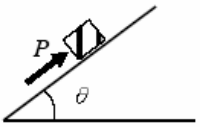
**ANSWER: B**

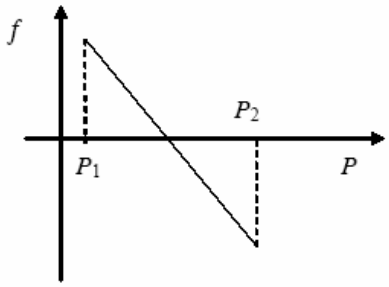
62. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is

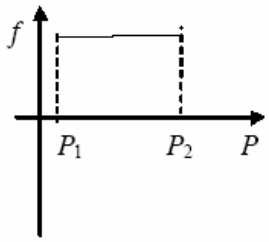
A) IBL  B) $\frac{IBL}{\pi}$  C) $\frac{IBL}{2\pi}$  D) $\frac{IBL}{4\pi}$

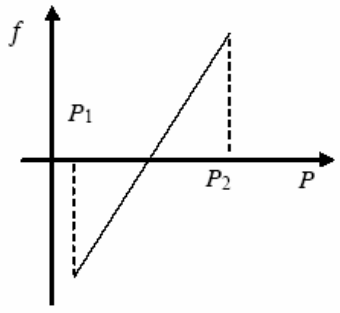
**ANSWER: C**

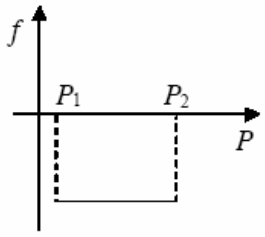
63. A block of mass m is on an inclined plane of angle $\theta$. The coefficient of friction between the block and the plane is $\mu$ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like




A)


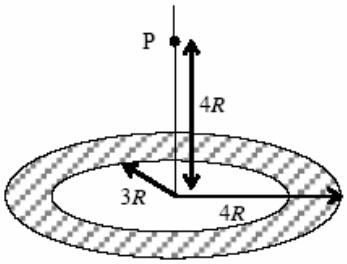

B)



C)



D)



**ANSWER: A**

64. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is

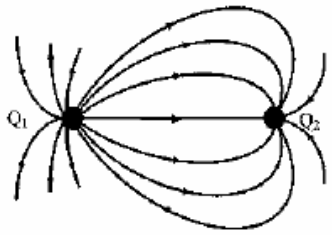



A) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ B) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ C) $\frac{GM}{4R}$ D) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$

**ANSWER: A**

## SECTION – II (Multiple Correct Choice Type)

65. A few electric field lines for a system of two charges $Q_1$ and $Q_2$ fixed at two different points on the x-axis are shown in the figure. These lines suggest that




A) $|Q_1| > |Q_2|$
B) $|Q_1| < |Q_2|$
C) at a finite distance to the left of $Q_1$ the electric field is zero
D) at a finite distance to the right of $Q_2$ the electric field is zero

**ANSWER: A and D**

66. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true ?

A) Error $\Delta T$ in measuring T, the time period, is 0.05 seconds
B) Error $\Delta T$ in measuring T, the time period, is 1 second
C) Percentage error in the determination of g is 5%
D) Percentage error in the determination of g is 2.5%

**ANSWER: A and C**

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67. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 $ms^{-1}$. Which of the following statement(s) is (are) correct for the system of these two masses ?

A) Total momentum of the system is 3 kg $ms^{-1}$

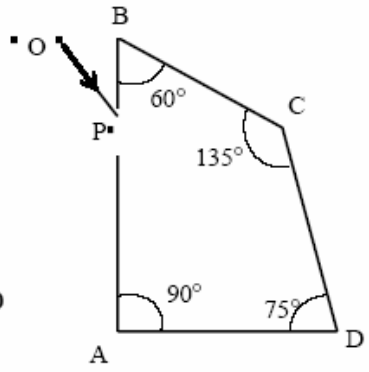
B) Momentum of 5 kg mass after collision is 4 kg $ms^{-1}$

C) Kinetic energy of the centre of mass is 0.75 J

D) Total kinetic energy of the system is 4 J

**ANSWER: A and C**

68. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of $60^0$ (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct ?




A) The ray gets totally internally reflected at face CD

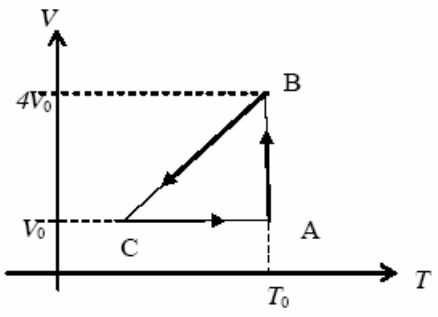
B) The ray comes out through face AD

C) The angle between the incident ray and the emergent ray is 90°

D) The angle between the incident ray and the emergent ray is 120°

**ANSWER: A and B and C**

69. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is $P_0$. Choose the correct option(s) from the following




A) Internal energies at A and B are the same

B) Work done by the gas in process AB is $P_0V_0 \ell n\ 4$

C) Pressure at C is $\frac{P_0}{4}$

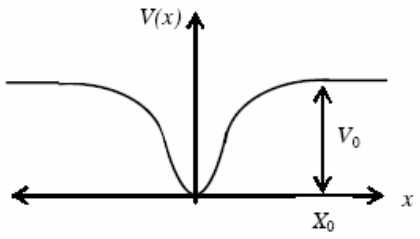
D) Temperature at C is $\frac{T_0}{4}$

**ANSWER: A and B and C and D**

## SECTION – III (Paragraph Type)

### Paragraph for Questions 70 to 72

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from $kx^2$ and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for $|x|$ near the origin and becomes a constant equal to $V_0$ for $|x| \geq X_0$ (see figure).




70. If the total energy of the particle is $E$, it will perform periodic motion only if

A) $E < 0$ B) $E > 0$ C) $V_0 > E > 0$ D) $E > V_0$

**ANSWER: B or C or (B and C)**
**Option C implies option B.**

71. For periodic motion of small amplitude $A$, the time period $T$ of this particle is proportional to

A) $A\sqrt{\frac{m}{\alpha}}$ B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ C) $A\sqrt{\frac{\alpha}{m}}$ D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

**ANSWER: B**

72. The acceleration of this particle for $|x| > X_0$ is

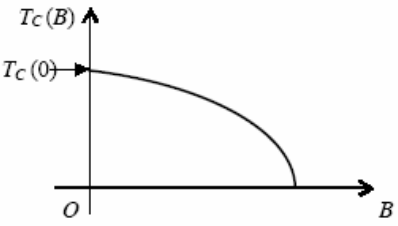
B) proportional to $\frac{V_0}{mX_0}$ A) proportional to $V_0$

C) proportional to $\sqrt{\frac{V_0}{mX_0}}$ D) zero

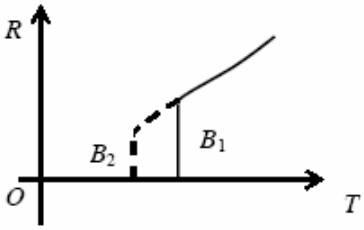
**ANSWER: D**

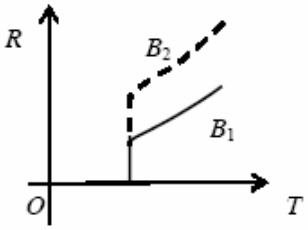
## Paragraph for Questions 73 to 74

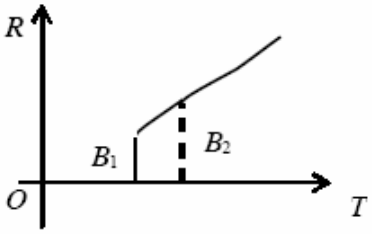
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_c(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_c(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_c(B)$ is a function of the magnetic field strength B. The dependence of $T_c(B)$ on B is shown in the figure.

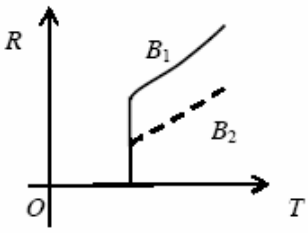



73. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields $B_1$ (solid line) and $B_2$ (dashed line). If $B_2$ is larger than $B_1$, which of the following graphs shows the correct variation of R with T in these fields ?

A)



B)



C)



D)



**ANSWER: A**

74. A superconductor has $T_c(0) = 100$ K. When a magnetic field of 7.5 Tesla is applied, its $T_c$ decreases to 75 K. For this material one can definitely say that when

A) B = 5 Tesla, $T_c(B) = 80$ K

B) B = 5 Tesla, 75 K < $T_c(B)$ < 100 K

C) B = 10 Tesla, 75 K < $T_c(B)$ < 100 K

D) B = 10 Tesla, $T_c(B) = 70$ K

**ANSWER: B**

## SECTION – IV (Integer Type)

75. The focal length of a thin biconvex lens is 20cm. When an object is moved from a distance of 25cm in front of it to 50cm, the magnification of its image changes from $m_{25}$ to $m_{50}$. The ratio $\frac{m_{25}}{m_{50}}$ is

**ANSWER: 6**

76. An $\alpha$-particle and a proton are accelerated from rest by a potential difference of 100V. After this, their de Broglie wavelengths are $\lambda_\alpha$ and $\lambda_p$ respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

**ANSWER: 3**

77. When two identical batteries of internal resistance $1\Omega$ each are connected in series across a resistor R, the rate of heat produced in R is $J_1$. When the same batteries are connected in parallel across R, the rate is $J_2$. If $J_1 = 2.25\ J_2$ then the value of R in $\Omega$ is

**ANSWER: 4**

78. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures $T_1$ and $T_2$, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B ?

**ANSWER: 9**

79. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

**ANSWER: 5**

80. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross-sectional area is $4.9\times10^{-7}m^2$. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad $s^{-1}$. If the Young's modulus of the material of the wire is $n\times10^9$ $Nm^{-2}$, the value of n is

**ANSWER: 4**

81. A binary star consists of two stars A (mass $2.2M_S$) and B (mass $11M_S$), where $M_S$ is the mass of the sun. They are separated by distance $d$ and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is

**ANSWER: 6**

82. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 $kms^{-1}$, the escape speed on the surface of the planet in $kms^{-1}$ will be

**ANSWER: 3**

83. A piece of ice (heat capacity = 2100 J $kg^{-1}$ $°C^{-1}$ and latent heat = $3.36\times10^5$ J $kg^{-1}$) of mass m grams is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

**ANSWER: 8**

84. A stationary source is emitting sound at a fixed frequency $f_0$, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of $f_0$. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 $ms^{-1}$.

**ANSWER: 7**

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