

PART I : CHEMISTRY

SECTION – I (Total Marks : 24)

(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Oxidation states of the metal in the minerals haematite and magnetite, respectively, are
- (A) II, III in haematite and III in magnetite
(B) II, III in haematite and II in magnetite
(C) II in haematite and II, III in magnetite
(D) III in haematite and II, III in magnetite

ANSWER : D

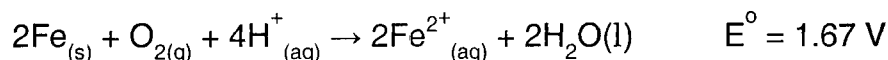
2. Among the following complexes (**K–P**),
 $K_3[Fe(CN)_6]$ (**K**), $[Co(NH_3)_6]Cl_3$ (**L**), $Na_3[Co(oxalate)_3]$ (**M**), $[Ni(H_2O)_6]Cl_2$ (**N**),
 $K_2[Pt(CN)_4]$ (**O**) and $[Zn(H_2O)_6](NO_3)_2$ (**P**)
the diamagnetic complexes are
- (A) **K, L, M, N** (B) **K, M, O, P** (C) **L, M, O, P** (D) **L, M, N, O**

ANSWER : C

3. Passing H_2S gas into a mixture of Mn^{2+} , Ni^{2+} , Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates
- (A) CuS and HgS (B) MnS and CuS
(C) MnS and NiS (D) NiS and HgS

ANSWER : A

4. Consider the following cell reaction:



At $[Fe^{2+}] = 10^{-3} \text{ M}$, $P(O_2) = 0.1 \text{ atm}$ and $pH = 3$, the cell potential at $25^{\circ}C$ is

- (A) 1.47 V (B) 1.77 V (C) 1.87 V (D) 1.57 V

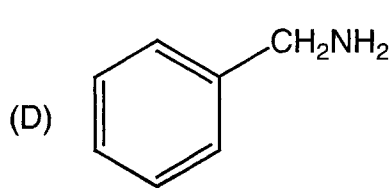
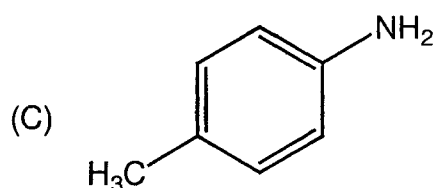
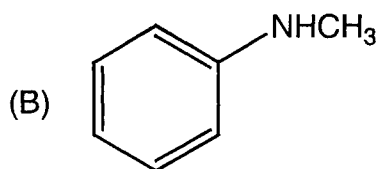
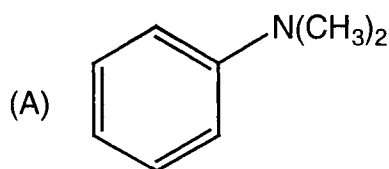
ANSWER : D

5. The freezing point (in $^{\circ}\text{C}$) of a solution containing 0.1 g of $\text{K}_3[\text{Fe}(\text{CN})_6]$ (Mol. Wt. 329) in 100 g of water ($K_f = 1.86 \text{ K kg mol}^{-1}$) is

(A) -2.3×10^{-2} (B) -5.7×10^{-2} (C) -5.7×10^{-3} (D) -1.2×10^{-2}

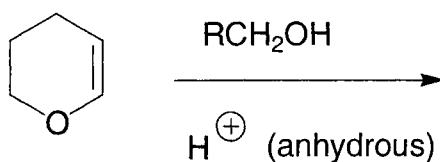
ANSWER: A

6. Amongst the compounds given, the one that would form a brilliant colored dye on treatment with NaNO_2 in dil. HCl followed by addition to an alkaline solution of β -naphthol is



ANSWER: C

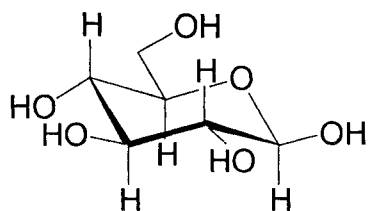
7. The major product of the following reaction is



- (A) a hemiacetal (B) an acetal
(C) an ether (D) an ester

ANSWER: B

8. The following carbohydrate is



- (A) a ketohexose (B) an aldohexose
(C) an α -furanose (D) an α -pyranose

ANSWER: B

SECTION – II (Total Marks : 16)**(Multiple Correct Answer(s) Type)**

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

9. Reduction of the metal centre in aqueous permanganate ion involves
- (A) 3 electrons in neutral medium (B) 5 electrons in neutral medium
(C) 3 electrons in alkaline medium (D) 5 electrons in acidic medium

ANSWER: ACD

10. The equilibrium

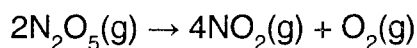


in aqueous medium at 25 °C shifts towards the left in the presence of

- (A) NO_3^- (B) Cl^- (C) SCN^- (D) CN^-

ANSWER: BCD

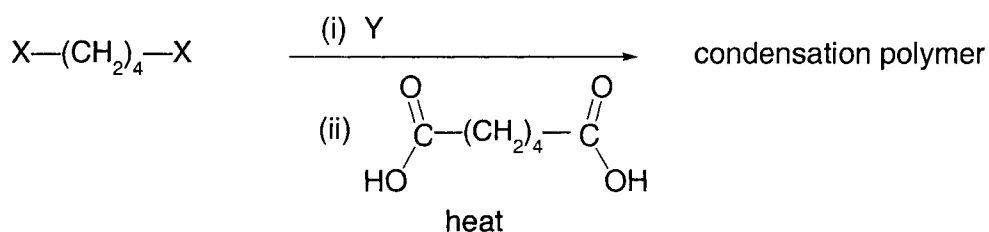
11. For the first order reaction



- (A) the concentration of the reactant decreases exponentially with time.
- (B) the half-life of the reaction decreases with increasing temperature.
- (C) the half-life of the reaction depends on the initial concentration of the reactant.
- (D) the reaction proceeds to 99.6 % completion in eight half-life duration.

ANSWER : ABD

12. The correct functional group X and the reagent/reaction conditions Y in the following scheme are



- (A) $\text{X} = \text{COOCH}_3$, $\text{Y} = \text{H}_2/\text{Ni}/\text{heat}$
- (B) $\text{X} = \text{CONH}_2$, $\text{Y} = \text{H}_2/\text{Ni}/\text{heat}$
- (C) $\text{X} = \text{CONH}_2$, $\text{Y} = \text{Br}_2/\text{NaOH}$
- (D) $\text{X} = \text{CN}$, $\text{Y} = \text{H}_2/\text{Ni}/\text{heat}$

ANSWER : CD

SECTION – III (Total Marks : 24)

(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

13. Among the following, the number of compounds than can react with PCl_5 to give POCl_3 is O_2 , CO_2 , SO_2 , H_2O , H_2SO_4 , P_4O_{10}

ANSWER: 4

14. The volume (in mL) of 0.1 M AgNO_3 required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[\text{Cr}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2$, as silver chloride is close to

ANSWER: 6

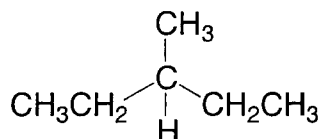
15. In 1 L saturated solution of AgCl [$K_{\text{sp}}(\text{AgCl}) = 1.6 \times 10^{-10}$], 0.1 mol of CuCl [$K_{\text{sp}}(\text{CuCl}) = 1.0 \times 10^{-6}$] is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x} . The value of "x" is

ANSWER: 7

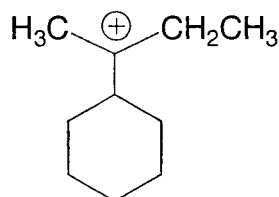
16. The number of hexagonal faces that are present in a truncated octahedron is

ANSWER: 8

17. The maximum number of isomers (including stereoisomers) that are possible on mono-chlorination of the following compound, is

**ANSWER: 8**

18. The total number of contributing structures showing hyperconjugation (involving C-H bonds) for the following carbocation is

**ANSWER: 6**

SECTION – IV (Total Marks : 16)

(Matrix-Match Type)

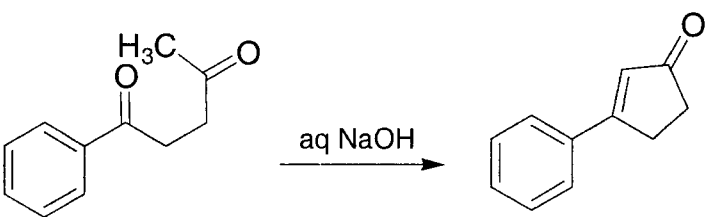
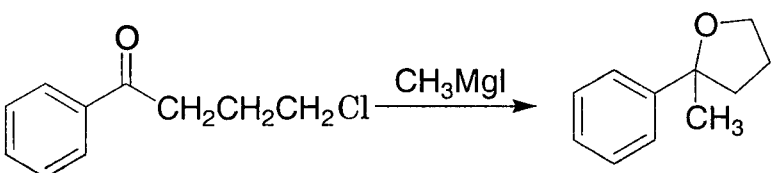
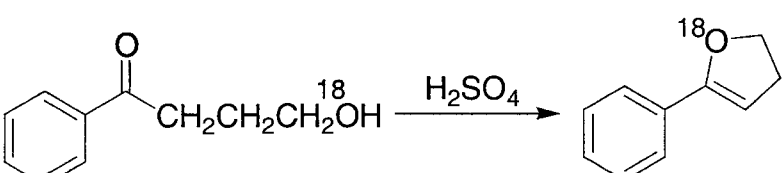
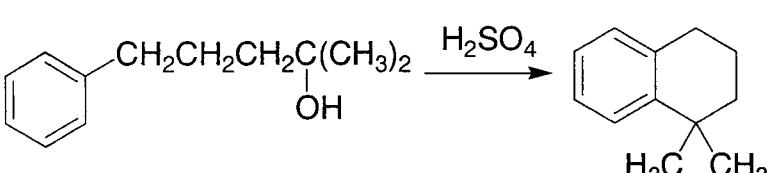
This section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

19. Match the transformations in **column I** with appropriate options in **column II**

Column I	Column II
(A) $\text{CO}_2(\text{s}) \rightarrow \text{CO}_2(\text{g})$	(p) phase transition
(B) $\text{CaCO}_3(\text{s}) \rightarrow \text{CaO}(\text{s}) + \text{CO}_2(\text{g})$	(q) allotropic change
(C) $2 \text{H}\cdot \rightarrow \text{H}_2(\text{g})$	(r) ΔH is positive
(D) $\text{P}_{(\text{white, solid})} \rightarrow \text{P}_{(\text{red, solid})}$	(s) ΔS is positive
	(t) ΔS is negative

ANSWER A : p, r and s**B : r and s****C : t****D : p, q and t**

20. Match the reactions in **column I** with appropriate types of steps/reactive intermediate involved in these reactions as given in **column II**

Column I	Column II
<p>(A) </p>	<p>(p) Nucleophilic substitution</p>
<p>(B) </p>	<p>(q) Electrophilic substitution</p>
<p>(C) </p>	<p>(r) Dehydration</p>
<p>(D) </p>	<p>(s) Nucleophilic addition</p>
	<p>(t) Carbanion</p>

ANSWER A : r, s and t

B : p and s

C : r and s

D : q and r

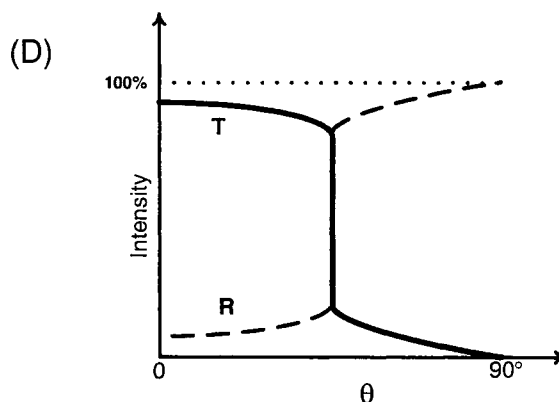
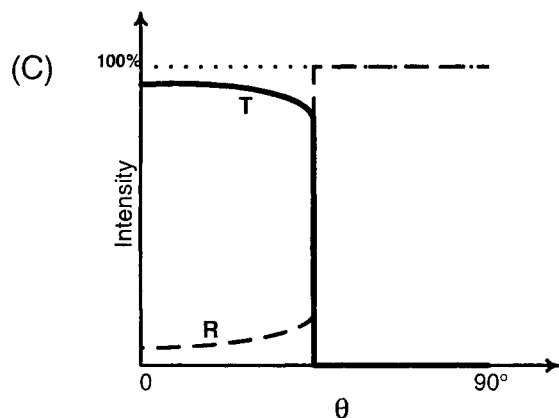
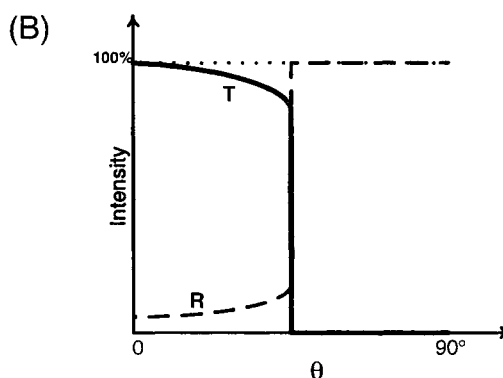
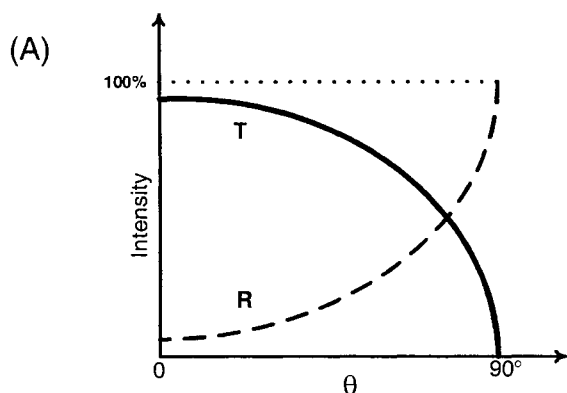
PART II : PHYSICS

SECTION – I (Total Marks : 24)

(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ . The reflected (R) and transmitted (T) intensities, both as function of θ , are plotted. The correct sketch is



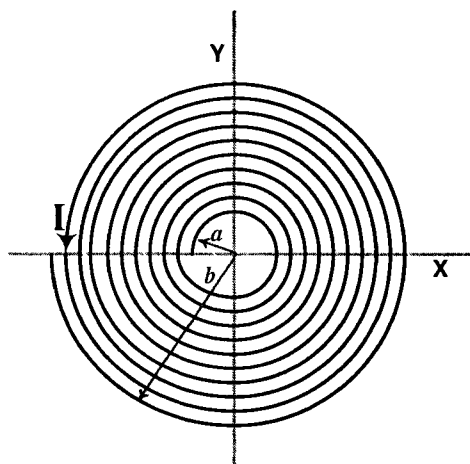
ANSWER : C

22. A satellite is moving with a constant speed ' V ' in a circular orbit about the earth. An object of mass ' m ' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

- (A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

ANSWER : B

23. A long insulated copper wire is closely wound as a spiral of ' N ' turns. The spiral has inner radius ' a ' and outer radius ' b '. The spiral lies in the X-Y plane and a steady current ' I ' flows through the wire. The Z-component of the magnetic field at the center of the spiral is



- (A) $\frac{\mu_0 N I}{2(b-a)} \ln\left(\frac{b}{a}\right)$ (B) $\frac{\mu_0 N I}{2(b-a)} \ln\left(\frac{b+a}{b-a}\right)$
 (C) $\frac{\mu_0 N I}{2b} \ln\left(\frac{b}{a}\right)$ (D) $\frac{\mu_0 N I}{2b} \ln\left(\frac{b+a}{b-a}\right)$

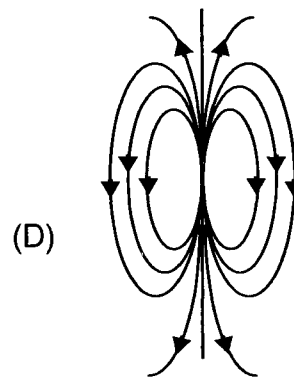
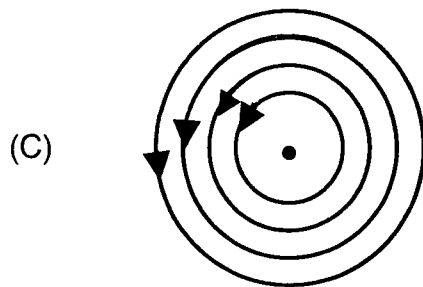
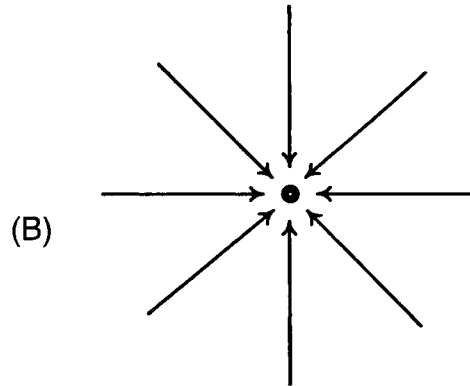
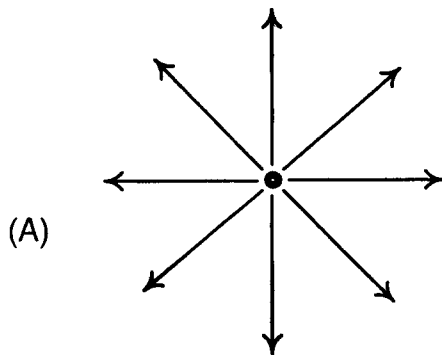
ANSWER : A

24. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

- (A) $\sqrt{2} A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3} A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

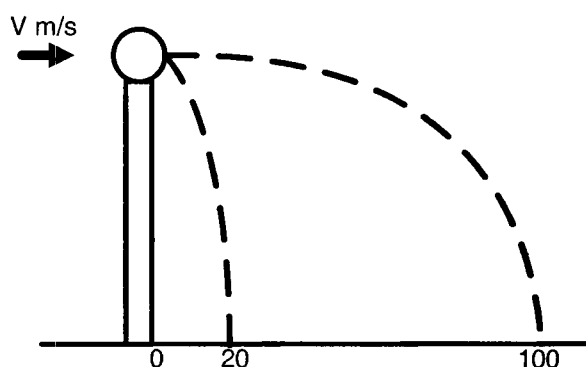
ANSWER : B

25. Which of the field patterns given below is valid for electric field as well as for magnetic field?



ANSWER : C

26. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is



- (A) 250 m/s (B) $250\sqrt{2}$ m/s (C) 400 m/s (D) 500 m/s

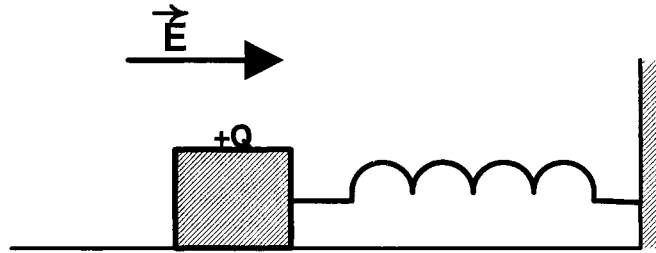
ANSWER : D

27. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

- (A) 0.9% (B) 2.4% (C) 3.1% (D) 4.2%

ANSWER : C

28. A wooden block performs SHM on a frictionless surface with frequency, ν_0 . The block carries a charge $+Q$ on its surface. If now a uniform electric field \vec{E} is switched-on as shown, then the SHM of the block will be



- (A) of the same frequency and with shifted mean position.
- (B) of the same frequency and with the same mean position.
- (C) of changed frequency and with shifted mean position.
- (D) of changed frequency and with the same mean position.

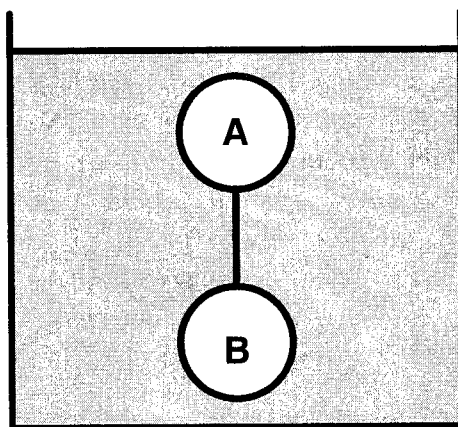
ANSWER : A

SECTION – II (Total Marks : 16)

(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

29. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F . They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



- (A) $d_A < d_F$ (B) $d_B > d_F$
 (C) $d_A > d_F$ (D) $d_A + d_B = 2 d_F$

ANSWER : ABD

30. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

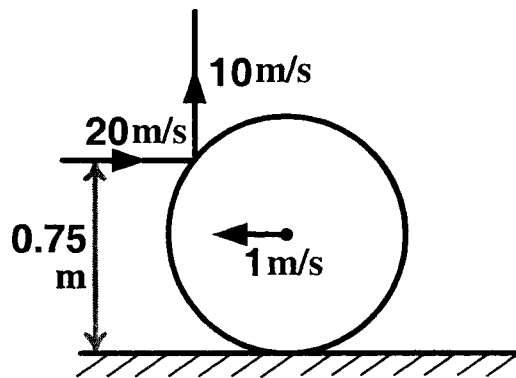
- (A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$
 (C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

ANSWER : BC

31. Which of the following statement(s) is/are correct?
- (A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2} , then the Gauss law will still be valid.
 - (B) The Gauss law can be used to calculate the field distribution around an electric dipole.
 - (C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
 - (D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$.

ANSWER : CD

32. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



- (A) the ring has pure rotation about its stationary CM.
- (B) the ring comes to a complete stop.
- (C) friction between the ring and the ground is to the left.
- (D) there is no friction between the ring and the ground.

ANSWER : A OR AC

SECTION – III (Total Marks : 24)

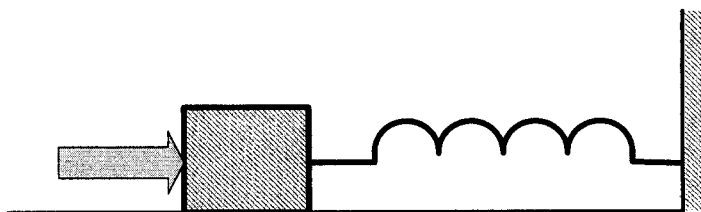
(Integer Answer Type)

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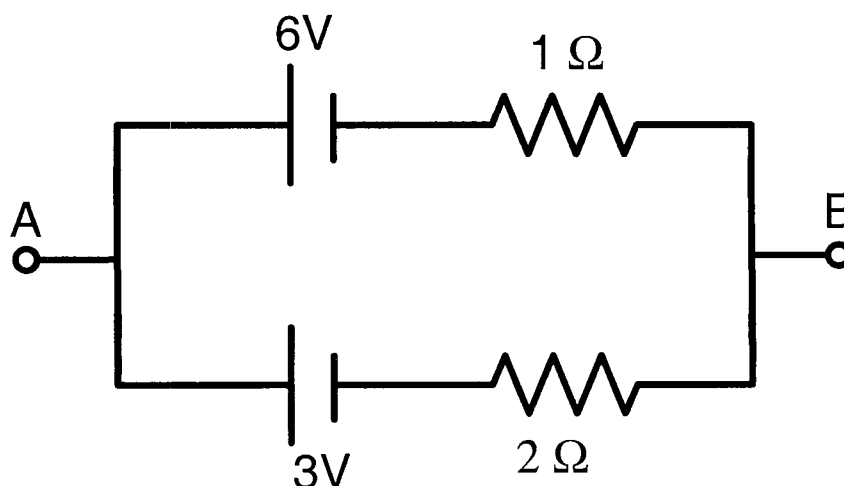
33. A train is moving along a straight line with a constant acceleration ' α '. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2 , is

ANSWER : 5

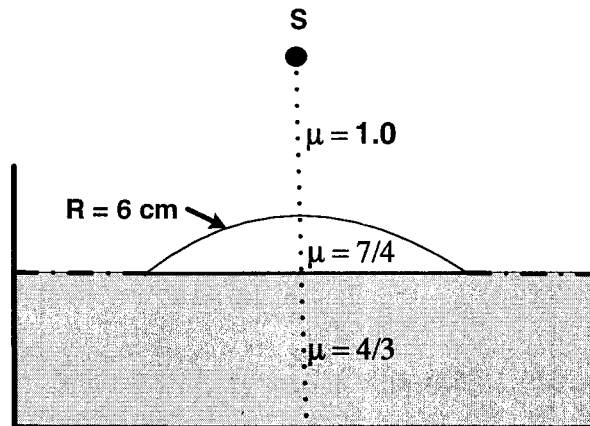
34. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $V = N/10$. Then N is

**ANSWER : 4**

35. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is

**ANSWER : 5**

36. Water (with refractive index $= \frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature ' $R = 6$ cm' as shown. Consider oil to act as a thin lens. An object ' S ' is placed 24 cm above water surface. The location of its image is at ' x ' cm above the bottom of the tank. Then ' x ' is



ANSWER : 2

37. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

ANSWER : 4

38. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of ' Z ' is

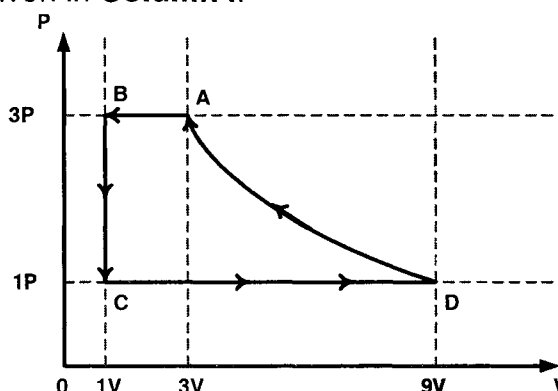
ANSWER : 7

SECTION – IV (Total Marks : 16)

(Matrix-Match Type)

This section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

39. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P-V diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I**.



Column I	Column II
(A) Process A \rightarrow B	(p) Internal energy decreases.
(B) Process B \rightarrow C	(q) Internal energy increases.
(C) Process C \rightarrow D	(r) Heat is lost.
(D) Process D \rightarrow A	(s) Heat is gained.
	(t) Work is done on the gas.

ANSWER A : p, r and t

B : p and r

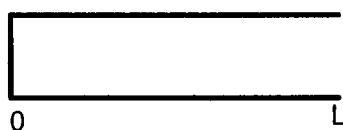
C : q and s

D : r and t

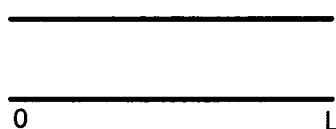
40. **Column I** shows four systems, each of the same length L , for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f . Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves.

Column I

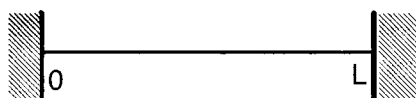
- (A) Pipe closed at one end



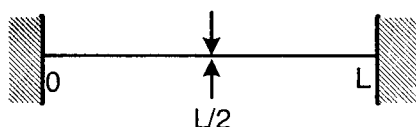
- (B) Pipe open at both ends



- (C) Stretched wire clamped at both ends



- (D) Stretched wire clamped at both ends and at mid-point



Column II

(p) Longitudinal waves

(q) Transverse waves

(r) $\lambda_f = L$

(s) $\lambda_f = 2L$

(t) $\lambda_f = 4L$

ANSWER A : p and t

B : p and s

C : q and s

D : q and r

PART III : MATHEMATICS

SECTION – I (Total Marks : 24)

(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

41. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x -axis at $(9, 0)$, then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

ANSWER : B

42. A value of b for which the equations

$$x^2 + bx - 1 = 0$$

$$x^2 + x + b = 0,$$

have one root in common is

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

ANSWER : B

43. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix},$$

where each of a , b , and c is either ω or ω^2 . Then the number of distinct matrices in the set S is

(A) 2 (B) 6 (C) 4 (D) 8

ANSWER : A

44. The circle passing through the point $(-1, 0)$ and touching the y -axis at $(0, 2)$ also passes through the point

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) $(-4, 0)$

ANSWER : D

45. If

$$\lim_{x \rightarrow 0} \left[1 + x \ln(1 + b^2) \right]^{\frac{1}{x}} = 2b \sin^2 \theta, \quad b > 0 \text{ and } \theta \in (-\pi, \pi],$$

then the value of θ is

- (A) $\pm \frac{\pi}{4}$ (B) $\pm \frac{\pi}{3}$ (C) $\pm \frac{\pi}{6}$ (D) $\pm \frac{\pi}{2}$

ANSWER : D

46. Let $f : [-1, 2] \rightarrow [0, \infty)$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1, 2]$.

Let $R_1 = \int_{-1}^2 x f(x) dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x -axis. Then

- (A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

ANSWER : C

47. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is

- (A) $\pm \sqrt{n\pi}$, $n \in \{0, 1, 2, \dots\}$ (B) $\pm \sqrt{n\pi}$, $n \in \{1, 2, \dots\}$
 (C) $\frac{\pi}{2} + 2n\pi$, $n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$ (D) $2n\pi$, $n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

ANSWER : A

48. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0, 0)$ to (x, y) in the ratio 1:3. Then the locus of P is

- (A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

ANSWER : C

SECTION – II (Total Marks : 16)

(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

49. If

$$f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ln x, & x > 1, \end{cases}$$

then

- (A) $f(x)$ is continuous at $x = -\frac{\pi}{2}$ (B) $f(x)$ is not differentiable at $x = 0$
 (C) $f(x)$ is differentiable at $x = 1$ (D) $f(x)$ is differentiable at $x = -\frac{3}{2}$

ANSWER : ABCD

50. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T , then

- (A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$
 (C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

ANSWER : AD

51. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then L is given by

- (A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$
 (C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

ANSWER : ABD

52. Let $f : (0, 1) \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{b-x}{1-bx},$$

where b is a constant such that $0 < b < 1$. Then

- (A) f is not invertible on $(0, 1)$
 (B) $f \neq f^{-1}$ on $(0, 1)$ and $f'(b) = \frac{1}{f'(0)}$
 (C) $f = f^{-1}$ on $(0, 1)$ and $f'(b) = \frac{1}{f'(0)}$
 (D) f^{-1} is differentiable on $(0, 1)$

ANSWER : A

SECTION – III (Total Marks : 24)

(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

53. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that

$$a + b + c = x$$

$$a + b\omega + c\omega^2 = y$$

$$a + b\omega^2 + c\omega = z.$$

Then the value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

ANSWER : MARKS TO ALL

54. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

ANSWER : 2

55. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in \mathbb{R}$, where $f'(x)$ denotes $\frac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on \mathbb{R} with $g(0) = g(2) = 0$. Then the value of $y(2)$ is

ANSWER : 0

56. Let M be a 3×3 matrix satisfying

$$M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \quad M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \text{and} \quad M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}.$$

Then the sum of the diagonal entries of M is

ANSWER : 9

57. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If \vec{r} is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then the value of $\vec{r} \cdot \vec{b}$ is

ANSWER : 9

58. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \leq 6$ into two parts. If

$$S = \left\{ \left(2, \frac{3}{4} \right), \left(\frac{5}{2}, \frac{3}{4} \right), \left(\frac{1}{4}, -\frac{1}{4} \right), \left(\frac{1}{8}, \frac{1}{4} \right) \right\},$$

then the number of point(s) in S lying inside the smaller part is

ANSWER : 2

SECTION – IV (Total Marks : 16)

(Matrix-Match Type)

This section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

59. Match the statements given in **Column I** with the values given in **Column II**

Column I	Column II
(A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between \vec{a} and \vec{b} is	(p) $\frac{\pi}{6}$
(B) If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q) $\frac{2\pi}{3}$
(C) The value of $\frac{\pi^2}{\ln 3} \int_{\frac{1}{6}}^{\frac{5}{6}} \sec(\pi x) dx$ is	(r) $\frac{\pi}{3}$
(D) The maximum value of $\left \text{Arg} \left(\frac{1}{1-z} \right) \right $ for $ z =1$, $z \neq 1$ is given by	(s) π
	(t) $\frac{\pi}{2}$

ANSWER A : q

B : p or p, q, r, s and t

C : s

D : t

60. Match the statements given in **Column I** with the intervals/union of intervals given in **Column II**

Column I

- (A) The set $\left\{ \operatorname{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number, } |z|=1, z \neq \pm 1 \right\}$ is
- (B) The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is
- (C) If $f(\theta) = \begin{vmatrix} 1 & \tan \theta & 1 \\ -\tan \theta & 1 & \tan \theta \\ -1 & -\tan \theta & 1 \end{vmatrix}$, then the set $\left\{ f(\theta) : 0 \leq \theta < \frac{\pi}{2} \right\}$ is
- (D) If $f(x) = x^{\frac{3}{2}}(3x-10)$, $x \geq 0$, then $f(x)$ is increasing in

Column II

- (p) $(-\infty, -1) \cup (1, \infty)$
- (q) $(-\infty, 0) \cup (0, \infty)$
- (r) $[2, \infty)$
- (s) $(-\infty, -1] \cup [1, \infty)$
- (t) $(-\infty, 0] \cup [2, \infty)$

ANSWER A : s

B : t

C : r

D : r

PART I : CHEMISTRY

SECTION – I (Total Marks : 21)

(Single Correct Answer Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. Extra pure N_2 can be obtained by heating

(A) NH_3 with CuO (B) NH_4NO_3
(C) $(NH_4)_2Cr_2O_7$ (D) $Ba(N_3)_2$

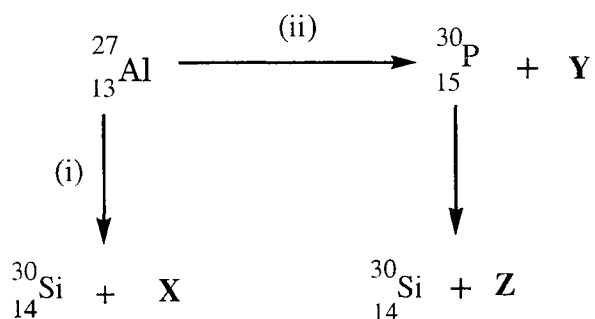
ANSWER: D

2. Geometrical shapes of the complexes formed by the reaction of Ni^{2+} with Cl^- , CN^- and H_2O , respectively, are

(A) octahedral, tetrahedral and square planar
(B) tetrahedral, square planar and octahedral
(C) square planar, tetrahedral and octahedral
(D) octahedral, square planar and octahedral

ANSWER: B

3. Bombardment of aluminum by α -particle leads to its artificial disintegration in two ways, (i) and (ii) as shown. Products **X**, **Y** and **Z** respectively are,



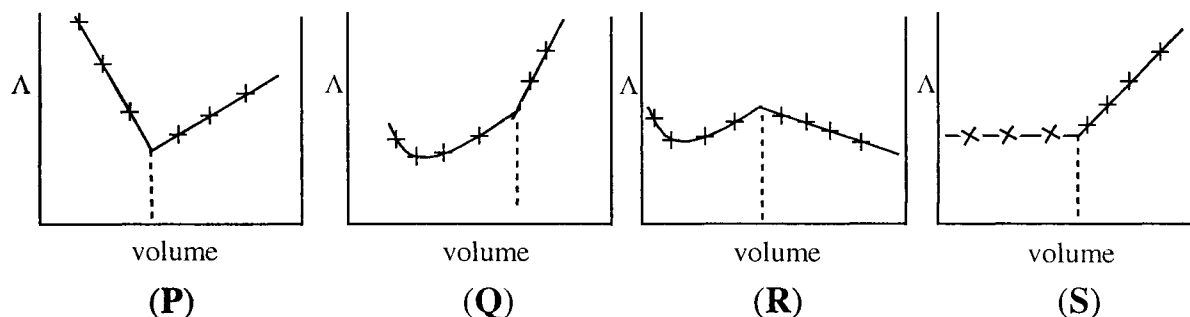
(A) proton, neutron, positron (B) neutron, positron, proton
(C) proton, positron, neutron (D) positron, proton, neutron

ANSWER: A

4. Dissolving 120 g of urea (mol. wt. 60) in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is
- (A) 1.78 M (B) 2.00 M (C) 2.05 M (D) 2.22 M

ANSWER: C

5. AgNO_3 (aq.) was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance (Λ) *versus* the volume of AgNO_3 is



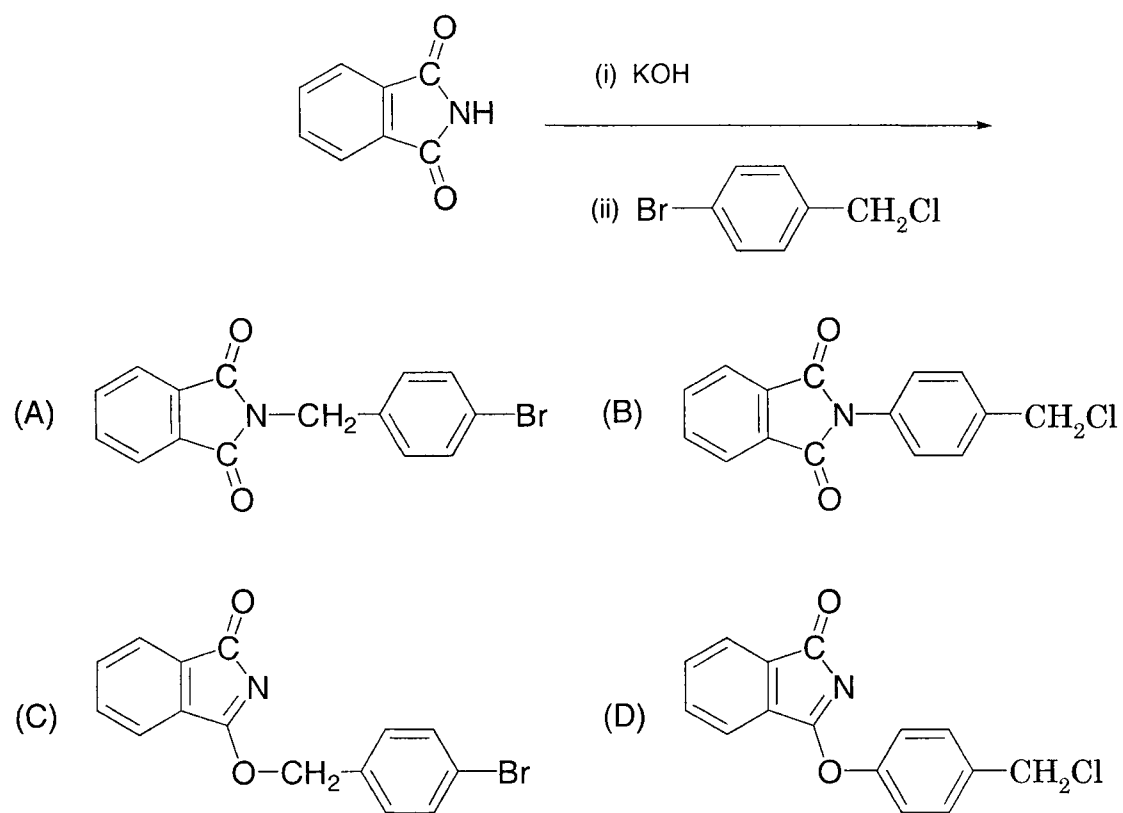
- (A) (P) (B) (Q) (C) (R) (D) (S)

ANSWER: D

6. Among the following compounds, the most acidic is
- (A) *p*-nitrophenol (B) *p*-hydroxybenzoic acid
 (C) *o*-hydroxybenzoic acid (D) *p*-toluic acid

ANSWER: C

7. The major product of the following reaction is



ANSWER: A

SECTION – II (Total Marks : 16)**(Multiple Correct Answers Type)**

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

8. Extraction of metal from the ore **cassiterite** involves
- (A) carbon reduction of an oxide ore (B) self-reduction of a sulphide ore
(C) removal of copper impurity (D) removal of iron impurity

ANSWER : AD

9. The correct statement(s) pertaining to the adsorption of a gas on a solid surface is (are)
- (A) Adsorption is always exothermic.
(B) Physisorption may transform into chemisorption at high temperature.
(C) Physisorption increases with increasing temperature but chemisorption decreases with increasing temperature.
(D) Chemisorption is more exothermic than physisorption, however it is very slow due to higher energy of activation.

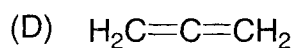
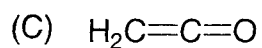
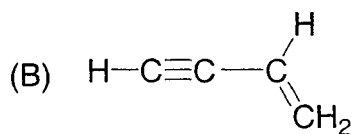
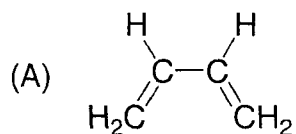
ANSWER : ABD

10. According to kinetic theory of gases

- (A) collisions are always elastic.
- (B) heavier molecules transfer more momentum to the wall of the container.
- (C) only a small number of molecules have very high velocity.
- (D) between collisions, the molecules move in straight lines with constant velocities.

ANSWER : ACD

11. Amongst the given options, the compound(s) in which all the atoms are in one plane in all the possible conformations (if any), is (are)



ANSWER : BC

SECTION – III (Total Marks : 15)**(Paragraph Type)**

This section contains **2 paragraphs**. Based upon one of the paragraphs **3 multiple choice questions** and based on the other paragraph **2 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 12 to 14

When a metal rod **M** is dipped into an aqueous colourless concentrated solution of compound **N**, the solution turns light blue. Addition of aqueous NaCl to the blue solution gives a white precipitate **O**. Addition of aqueous NH_3 dissolves **O** and gives an intense blue solution.

12. The metal rod **M** is

- (A) Fe (B) Cu (C) Ni (D) Co

ANSWER : B

13. The compound **N** is

- (A) AgNO_3 (B) $\text{Zn(NO}_3)_2$
(C) $\text{Al(NO}_3)_3$ (D) $\text{Pb(NO}_3)_2$

ANSWER : A

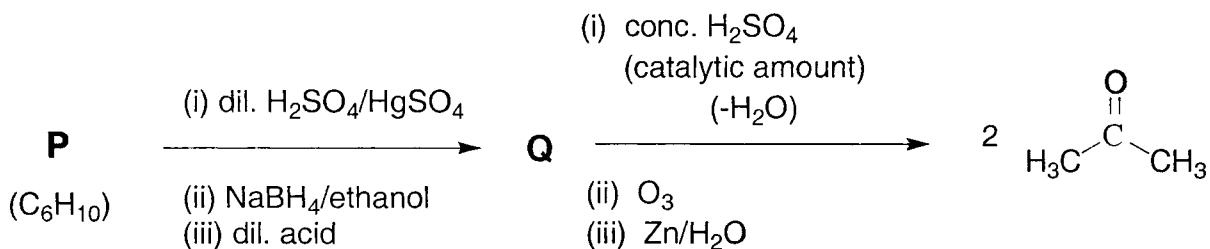
14. The final solution contains

- (A) $[\text{Pb(NH}_3)_4]^{2+}$ and $[\text{CoCl}_4]^{2-}$ (B) $[\text{Al(NH}_3)_4]^{3+}$ and $[\text{Cu(NH}_3)_4]^{2+}$
(C) $[\text{Ag(NH}_3)_2]^+$ and $[\text{Cu(NH}_3)_4]^{2+}$ (D) $[\text{Ag(NH}_3)_2]^+$ and $[\text{Ni(NH}_3)_6]^{2+}$

ANSWER : C

Paragraph for Question Nos. 15 and 16

An acyclic hydrocarbon **P**, having molecular formula C_6H_{10} , gave acetone as the only organic product through the following sequence of reactions, in which **Q** is an intermediate organic compound.



15. The structure of compound **P** is

- (A) $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2-\text{C}\equiv\text{C}-\text{H}$ (B) $\text{H}_3\text{CH}_2\text{C}-\text{C}\equiv\text{C}-\text{CH}_2\text{CH}_3$
- (C) $\begin{array}{c} \text{H}_3\text{C} \\ | \\ \text{H}-\text{C}-\text{C}\equiv\text{C}-\text{CH}_3 \\ | \\ \text{H}_3\text{C} \end{array}$ (D) $\begin{array}{c} \text{H}_3\text{C} \\ | \\ \text{H}_3\text{C}-\text{C}-\text{C}\equiv\text{C}-\text{H} \\ | \\ \text{H}_3\text{C} \end{array}$

ANSWER : D

16. The structure of the compound **Q** is

- (A) $\begin{array}{c} \text{H}_3\text{C} \quad \text{OH} \\ | \quad | \\ \text{H}-\text{C}-\text{C}-\text{CH}_2\text{CH}_3 \\ | \quad | \\ \text{H}_3\text{C} \quad \text{H} \end{array}$ (B) $\begin{array}{c} \text{H}_3\text{C} \quad \text{OH} \\ | \quad | \\ \text{H}_3\text{C}-\text{C}-\text{C}-\text{CH}_3 \\ | \quad | \\ \text{H}_3\text{C} \quad \text{H} \end{array}$
- (C) $\begin{array}{c} \text{H}_3\text{C} \quad \text{OH} \\ | \quad | \\ \text{H}-\text{C}-\text{CH}_2\text{CHCH}_3 \\ | \quad | \\ \text{H}_3\text{C} \quad \text{H} \end{array}$ (D) $\text{CH}_3\text{CH}_2\text{CH}_2\overset{\text{OH}}{\text{CH}}\text{CH}_2\text{CH}_3$

ANSWER : B

SECTION – IV (Total Marks : 28)**(Integer Answer Type)**

This section contains **7 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

17. The difference in the oxidation numbers of the two types of sulphur atoms in $\text{Na}_2\text{S}_4\text{O}_6$ is

ANSWER : 5

18. Reaction of Br_2 with Na_2CO_3 in aqueous solution gives sodium bromide and sodium bromate with evolution of CO_2 gas. The number of sodium bromide molecules involved in the balanced chemical equation is

ANSWER : 5

19. The maximum number of electrons that can have principal quantum number, $n = 3$, and spin quantum number, $m_s = -\frac{1}{2}$, is

ANSWER : 9

20. The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

ANSWER: 4

21. To an evacuated vessel with movable piston under external pressure of 1 atm., 0.1 mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm. at 0 °C) are introduced. Considering the ideal gas behaviour, the total volume (in litre) of the gases at 0 °C is close to

ANSWER: 7

22. The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is

ANSWER: 5

23. A decapeptide (Mol. Wt. 796) on complete hydrolysis gives glycine (Mol. Wt. 75), alanine and phenylalanine. Glycine contributes 47.0 % to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is

ANSWER: 6

PART II : PHYSICS

SECTION – I (Total Marks : 21)

(Single Correct Answer Type)

This section contains 7 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

24. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

(A) 8.50 kHz (B) 8.25 kHz (C) 7.75 kHz (D) 7.50 kHz

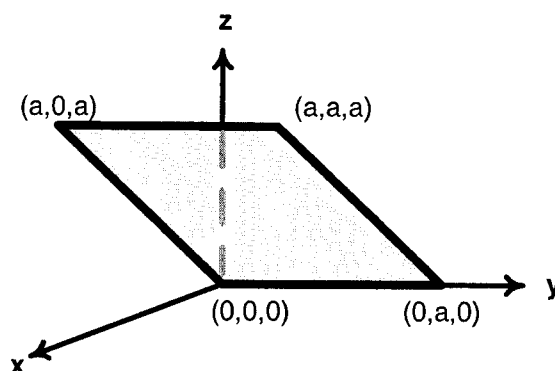
ANSWER: A

25. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1 , the work done in the process is

(A) $\frac{9}{8} R T_1$ (B) $\frac{3}{2} R T_1$ (C) $\frac{15}{8} R T_1$ (D) $\frac{9}{2} R T_1$

ANSWER: A

26. Consider an electric field $\vec{E} = E_0 \hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(A) $2E_0 a^2$ (B) $\sqrt{2} E_0 a^2$ (C) $E_0 a^2$ (D) $\frac{E_0 a^2}{\sqrt{2}}$

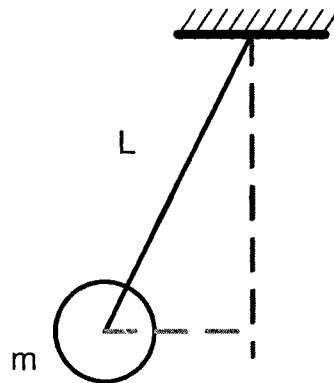
ANSWER: C

27. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 \AA . The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is

(A) 1215 \AA (B) 1640 \AA (C) 2430 \AA (D) 4687 \AA

ANSWER : A

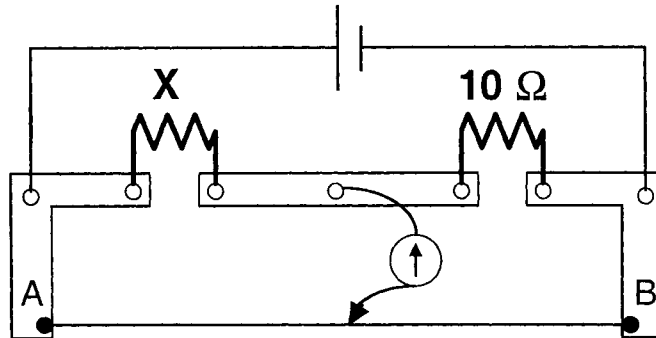
28. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m . The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N . The maximum possible value of angular velocity of ball (in radian/s) is



(A) 9 (B) 18 (C) 27 (D) 36

ANSWER : D

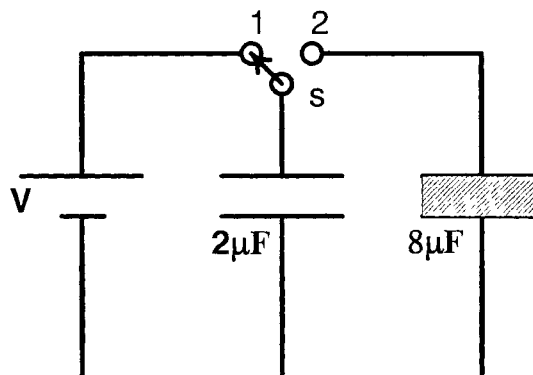
29. A meter bridge is set-up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is



- (A) 10.2 ohm (B) 10.6 ohm (C) 10.8 ohm (D) 11.1 ohm

ANSWER: B

30. A $2\ \mu\text{F}$ capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



- (A) 0% (B) 20% (C) 75% (D) 80%

ANSWER: D

SECTION – II (Total Marks : 16)**(Multiple Correct Answers Type)**

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

31. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge '+Q'. Now they are connected by a thin metal wire. Then

(A) $E_A^{inside} = 0$

(B) $Q_A > Q_B$

(C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

(D) $E_A^{on\ surface} < E_B^{on\ surface}$

ANSWER : ABCD

32. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?

(A) They will never come out of the magnetic field region.

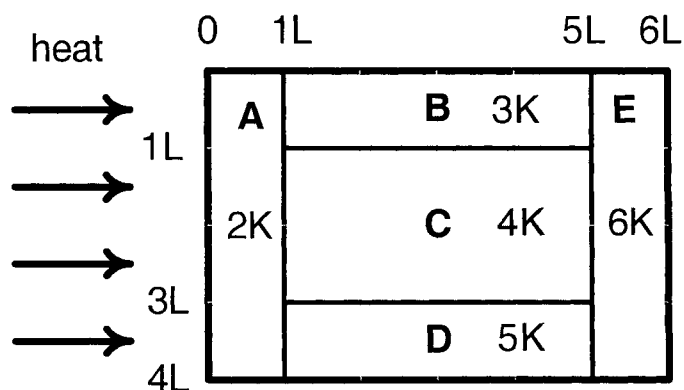
(B) They will come out travelling along parallel paths.

(C) They will come out at the same time.

(D) They will come out at different times.

ANSWER : BC, BD, BCD

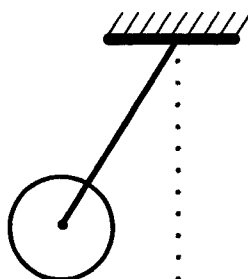
33. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat ' Q ' flows only from left to right through the blocks. Then in steady state



- (A) heat flow through A and E slabs are same.
 (B) heat flow through slab E is maximum.
 (C) temperature difference across slab E is smallest.
 (D) heat flow through C = heat flow through B + heat flow through D.

ANSWER: ACD

34. A metal rod of length ' L ' and mass ' m ' is pivoted at one end. A thin disk of mass ' M ' and radius ' R ' ($< L$) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A) The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disk system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true?



- (A) Restoring torque in case A = Restoring torque in case B
 (B) Restoring torque in case A < Restoring torque in case B
 (C) Angular frequency for case A > Angular frequency for case B.
 (D) Angular frequency for case A < Angular frequency for case B.

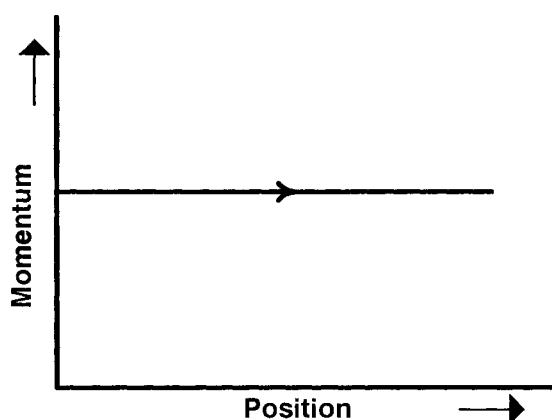
ANSWER: AD

SECTION – III (Total Marks : 15)**(Paragraph Type)**

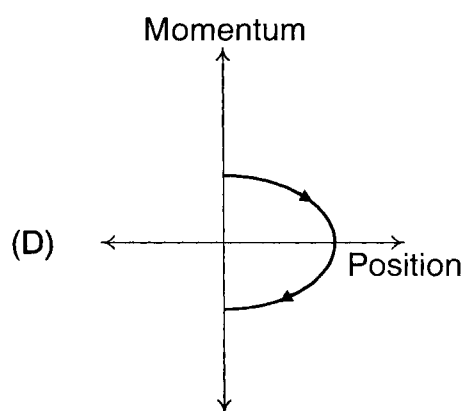
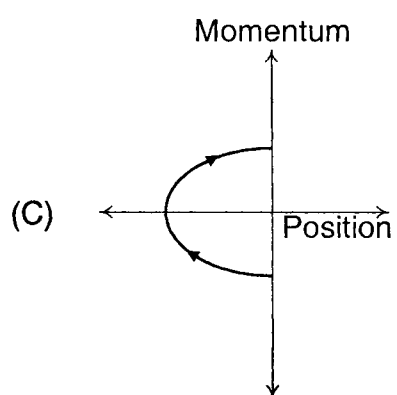
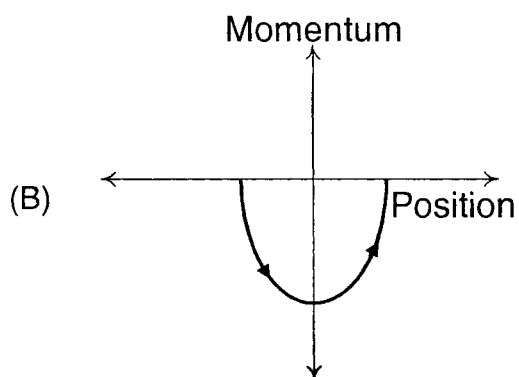
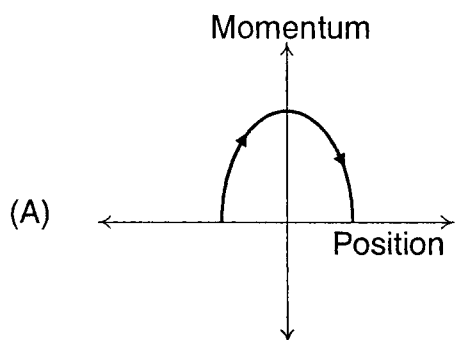
This section contains **2 paragraphs**. Based upon one of the paragraphs **3 multiple choice questions** and based on the other paragraph **2 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 35 to 37

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is $x(t)$ vs. $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.



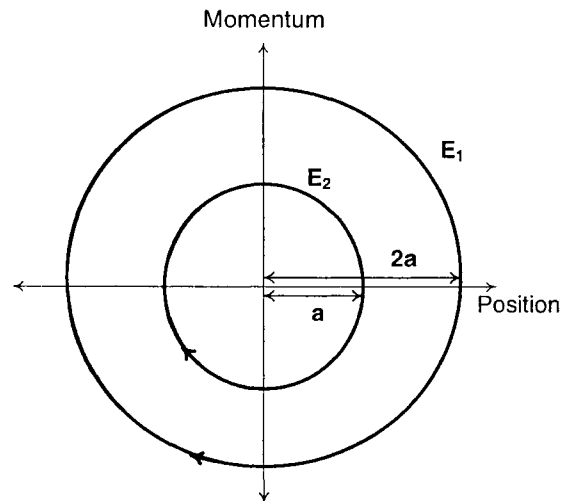
35. The phase space diagram for a ball thrown vertically up from ground is



ANSWER: D

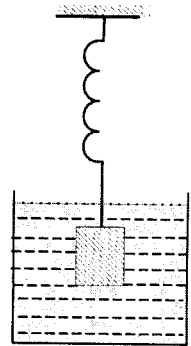
36. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

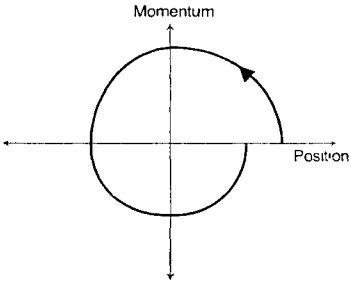
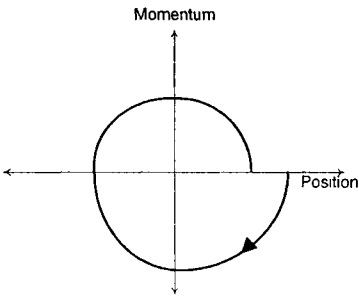
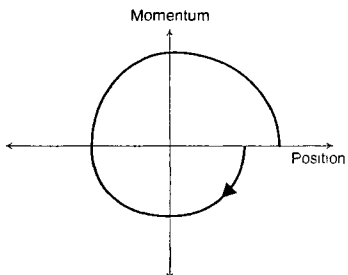
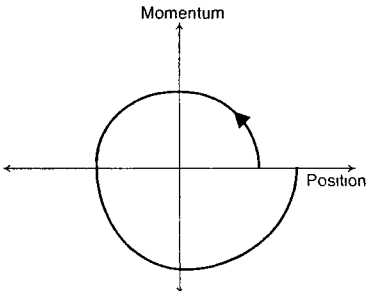
- (A) $E_1 = \sqrt{2} E_2$ (B) $E_1 = 2 E_2$
 (C) $E_1 = 4 E_2$ (D) $E_1 = 16 E_2$



ANSWER : C

37. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



- (A)  (B) 
 (C)  (D) 

ANSWER : B

Paragraph for Question Nos. 38 and 39

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ' ω_p ', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω , where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p , all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

38. Taking the electronic charge as ' e ' and the permittivity as ' ϵ_0 ', use dimensional analysis to determine the correct expression for ω_p .

(A) $\sqrt{\frac{Ne}{m\epsilon_0}}$ (B) $\sqrt{\frac{m\epsilon_0}{Ne}}$ (C) $\sqrt{\frac{Ne^2}{m\epsilon_0}}$ (D) $\sqrt{\frac{m\epsilon_0}{Ne^2}}$

ANSWER : C

39. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27} \text{ m}^{-3}$. Take $\epsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

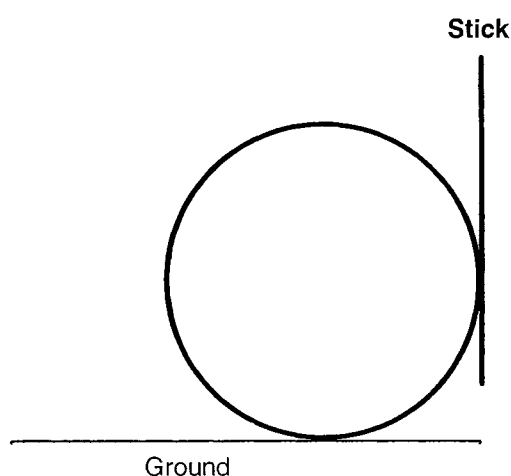
(A) 800 nm (B) 600 nm (C) 300 nm (D) 200 nm

ANSWER : B

SECTION – IV (Total Marks : 28)**(Integer Answer Type)**

This section contains **7 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

40. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2 . The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is $(P/10)$. The value of P is

**ANSWER : 4**

41. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ . The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10 \mu$, then N is

ANSWER : 5

42. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side ' a '. The surface tension of the soap film is γ . The system of charges and planar film are in equilibrium, and $a = k \left[\frac{q^2}{\gamma} \right]^{1/N}$, where ' k ' is a constant. Then N is

ANSWER: 3

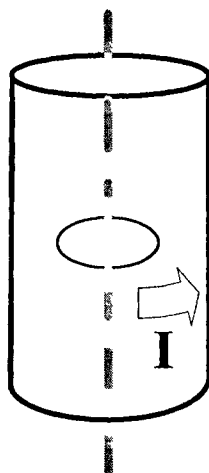
43. Steel wire of length ' L ' at 40°C is suspended from the ceiling and then a mass ' m ' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length ' L '. The coefficient of linear thermal expansion of the steel is $10^{-5} / ^\circ\text{C}$, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that $L \gg$ diameter of the wire. Then the value of ' m ' in kg is nearly

ANSWER: 3

44. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s . The mass of an atom of this radioisotope is 10^{-25} kg . The mass (in mg) of the radioactive sample is

ANSWER: 1

45. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N \mu_0 I_0 \sin(300t)$, then 'N' is



ANSWER: 6

46. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4} \text{ kg-m}^2$, then N is

ANSWER: 9

PART III : MATHEMATICS

SECTION – I (Total Marks : 21)

(Single Correct Answer Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

47. Let (x_0, y_0) be the solution of the following equations

$$\begin{aligned}(2x)^{\ln 2} &= (3y)^{\ln 3} \\ 3^{\ln x} &= 2^{\ln y}.\end{aligned}$$

Then x_0 is

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

ANSWER : C

48. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

- (A) $\frac{1}{4} \ln \frac{3}{2}$ (B) $\frac{1}{2} \ln \frac{3}{2}$
(C) $\ln \frac{3}{2}$ (D) $\frac{1}{6} \ln \frac{3}{2}$

ANSWER : A

49. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector \vec{v} in the plane of \vec{a} and \vec{b} , whose projection on \vec{c} is $\frac{1}{\sqrt{3}}$, is given by

- (A) $\hat{i} - 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} - \hat{k}$
 (C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

ANSWER: C

50. Let $P = \{\theta: \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$ and $Q = \{\theta: \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$ be two sets. Then

- (A) $P \subset Q$ and $Q - P \neq \emptyset$ (B) $Q \not\subset P$
 (C) $P \not\subset Q$ (D) $P = Q$

ANSWER: D

51. Let the straight line $x = b$ divide the area enclosed by $y = (1-x)^2$, $y = 0$, and $x = 0$ into two parts R_1 ($0 \leq x \leq b$) and R_2 ($b \leq x \leq 1$) such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

- (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

ANSWER: B

52. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

(A) 1 (B) 2 (C) 3 (D) 4

ANSWER: C

53. A straight line L through the point $(3, -2)$ is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x -axis, then the equation of L is

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
 (C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

ANSWER: B

SECTION – II (Total Marks : 16)

(Multiple Correct Answers Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

54. The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is/are

(A) $\hat{j} - \hat{k}$ (B) $-\hat{i} + \hat{j}$ (C) $\hat{i} - \hat{j}$ (D) $-\hat{j} + \hat{k}$

ANSWER: AD

55. Let M and N be two 3×3 non-singular skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P , then $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$ is equal to

(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

ANSWER: MARKS TO ALL

56. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(B) a focus of the hyperbola is $(2, 0)$

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

ANSWER: BD

57. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that

$$f(x+y) = f(x) + f(y), \quad \forall x, y \in \mathbb{R}.$$

If $f(x)$ is differentiable at $x = 0$, then

- (A) $f(x)$ is differentiable only in a finite interval containing zero
- (B) $f(x)$ is continuous $\forall x \in \mathbb{R}$
- (C) $f'(x)$ is constant $\forall x \in \mathbb{R}$
- (D) $f(x)$ is differentiable except at finitely many points

ANSWER: BC, BCD

SECTION – III (Total Marks : 15)

(Paragraph Type)

This section contains **2 paragraphs**. Based upon one of the paragraphs **3 multiple choice questions** and based on the other paragraph **2 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 58 to 60

Let a , b and c be three real numbers satisfying

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad \dots\dots\dots(E)$$

58. If the point $P(a, b, c)$, with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(A) 0 (B) 12 (C) 7 (D) 6

ANSWER: D

59. Let ω be a solution of $x^3 - 1 = 0$ with $\text{Im}(\omega) > 0$. If $a = 2$ with b and c satisfying (E), then the value of

$$\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$$

is equal to

(A) -2 (B) 2 (C) 3 (D) -3

ANSWER: A

60. Let $b = 6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta} \right)^n$$

is

(A) 6 (B) 7 (C) $\frac{6}{7}$ (D) ∞

ANSWER: B

Paragraph for Question Nos. 61 and 62

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2 . However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2 . Now 1 ball is drawn at random from U_2 .

61. The probability of the drawn ball from U_2 being white is

- (A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

ANSWER: B

62. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

- (A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

ANSWER: D

SECTION – IV (Total Marks : 28)

(Integer Answer Type)

This section contains **7 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

63. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

ANSWER: 2

64. Let $a_1, a_2, a_3, \dots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^p a_i$, $1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n , then a_2 is

ANSWER: 3, 9, 3 & 9 BOTH

65. The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$

is

ANSWER: 7

66. Let $f : [1, \infty) \rightarrow [2, \infty)$ be a differentiable function such that $f(1) = 2$. If

$$6 \int_1^x f(t) dt = 3x f(x) - x^3$$

for all $x \geq 1$, then the value of $f(2)$ is

ANSWER : MARKS TO ALL

67. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$, then the minimum value of $|2z - 6 + 5i|$ is

ANSWER : 5

68. The minimum value of the sum of real numbers a^{-5} , a^{-4} , $3a^{-3}$, 1 , a^8 and a^{10} with $a > 0$ is

ANSWER : 8

69. Let $f(\theta) = \sin \left(\tan^{-1} \left(\frac{\sin \theta}{\sqrt{\cos 2\theta}} \right) \right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of

$$\frac{d}{d(\tan \theta)} (f(\theta))$$

is

ANSWER : 1